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Abstract

This paper discusses the role of reserve prices when the signal of each bidder is positively affiliated with the seller's signal. We distinguish three reserve price designs: a public reserve price, announced before the auction starts, a revealed reserve price, disclosed when a bid matches it, and a secret reserve price that is disclosed after the highest bid has been reached. We show that a public or a revealed reserve price are strategically equivalent, and we show that no seller will set a secret reserve price.

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1 Introduction

It is a well-known result that affiliation among bidders' signals leads to a clear expected revenue ranking: the (English) ascending bid auction dominates the second-price auction, and the second-price auction dominates the first-price auction (see, for example, Krishna, 2002, Chapter 6). This is due to the linkage principle: the larger the linkage between the information the bidder receives by her private signal and how she thinks other bidders will behave, the larger is the expected winning bid. A rather under-researched topic is the case of affiliation between each bidder's and the seller's signals. The seller receives a certain signal for the item for sale, and this information may be more important than any signal another bidder receives.

In this paper, we scrutinize the role of affiliation between the signals of the seller and the bidders and its implication for the optimal reserve price. In order to focus on this aspect, we assume that the valuation of bidders does not depend on the signals other bidders receive, but only on the bidder's signal and the seller's signal. The seller's valuation depends only on his signal. How can sellers potentially reveal their information? In the case of affiliation among potential buyers, bidders learn other bidders' signals in an ascending auction only by observing their behavior, that is, when they drop out of the auction. In case of affiliation between bidders and the seller, they can only learn the seller's signal if the seller has set a reserve price that is either publicly announced before the auction or revealed when the recent bid has reached the reserve price. We will call the first case a *public* reserve price and the second case a *revealed* reserve price. The third case is the case of a *secret* reserve price that is disclosed only once the auction is over and the highest bid is realized. Our paper shows that the secret reserve price is dominated by the other two designs.

The remainder of this paper is organized as follows. Section 2 discusses the relevant literature, and section 3 introduces the model and shows the results for public and revealed reserve prices in ascending bid auctions. Section 4 discusses secret reserve prices and shows how our results extend to first- and second-price auctions. Section 5 concludes.

2 Related literature

The role of public reserve prices in standard auctions with independent private values is well established in the literature: when bidders are symmetric and risk-neutral, the standard auctions (i.e., the first-price and the second-price sealed-bid auctions, and the English auction) are optimal and the revenue-maximizing seller should set a public reserve price that exceeds her value; and this optimal reserve price does not depend on the number of bidders (Krishna, 2002). The main idea is the interplay between excluding bidders with values less than the reserve price and the risk that the highest value among all bidders might be less than the reserve price. Elyakime et al. (1994) - motivated by the practice in France to sell timber - extend this result to secret reserve prices: within the independent private values framework, they solve a first-price sealed-bid auction with symmetric and risk-neutral bidders for the optimal bidding and selling strategies, and they show that the optimal secret reserve price is the one that is equal to the seller's private value, and that the seller's expected gain increases by moving from an optimal secret reserve price to an optimal public reserve price.[1]

Relaxing some of the assumptions of the standard auctions, the literature has scrutinized the role of secret reserve prices in different environments. For instance, within the framework of independent private values, setting a secret reserve price may emerge as the equilibrium behavior in first-price auctions if there is a large number of bidders that are sufficiently risk averse and they respond to the secret reserve price by bidding more aggressively (as in Li and Tan, 2017). Rosar (2004) shows that reserve prices may dominate if the seller is risk-averse and the seller's information (for some exogenous reason) might improve after having committed to the rules of the auction and before the auction has started. Rosenkranz and Schmitz (2007) discuss the role of a secret reserve price in both first-price and second-price auctions if bidders have reference-based utility and a public reserve price partly serves as the reference point.[2]

[1] Also see Li and Perrigne (2003) and Elyakime and Loisel (2005) for similar results in support of the use public reserve prices in first-price auctions with independent private values.

[2] There are different strands of the auction literature also looking at the optimal reserve price policy, especially (i) when there is endogenous participation (competitive auctions), in which case depending on the specification of the competitive environment and the bidders' perception of the variation in participation rates, secret reserve prices may be preferred (as in Jehiel and Lamy, 2015); or (ii) when re-auctioning (or re-negotiating to sell) the good is possible, in which case depending on the characteristics

In general, within the framework of independent private values, the assumption of risk-averse bidders, per se, does not make much difference in ascending-bid auctions (where the bidders stay active so long as the current price is below their valuation), or in second-price auctions (where it is a weakly dominant strategy to bid one's true value); see, for example, Li and Tan (2017). Relaxing the assumption of independent private values, however, may generate incentives for the seller to set a secret reserve price. Vincent (1995) shows that a seller (privately knowing the value of the good) may increase the participation rate by keeping the reserve price secret in a second-price auction with affiliated values, and this increase in participation may increase the seller's ex-ante expected revenues. This result, however, relies on the implicit assumption that there is full commitment to secrecy and to (not) sell the good if the price is (below) above the secret reserve price. In the framework of second-price auctions with common (affiliated) values, Horstmann and LaCasse (1997) show that such a commitment may not be self-enforcing: if the seller has valuable private information about the value of the good, which he cannot directly communicate to the bidders, then he may prefer to set a secret reserve price and delay the sale of the good by refusing to sell (even for a high price) and by re-auctioning the good, especially if this generates a credible signal.

There is a rather limited number of studies looking at the role of reserve prices in ascending-bid auctions, most of them in the framework of independent private values for goods for which some post-auction resale market exists (where the good is sold either via another round of an auction or by negotiations); see, for example, Menezes and Ryan (2005) and Grant et al. (2006). While ascending-bid auctions are one of the most commonly observed auction format in different markets, affiliated values are also empirically relevant; see Milgrom and Weber (1982). As Cassady (1967) points out, a seller would know the particulars of the good (especially from past use and expert opinions) before the auction is run. Similarly, each bidder will receive some signal, and if the consumption of the good is purely a personal matter, bidders will consider only their own signal and any signal they receive from the seller. Then, as in our model, the valuation of bidders should depend only on their own signal and the seller's signal; see, inter alia, Milgrom and Weber (1982), and Horstmann and LaCasse (1997).

of the resale market (and the bargaining power of the agents), public reserve prices may emerge as the equilibrium policy (as in Menezes and Ryan, 2005, and Grant et al., 2006).

While Vincent (1995) has an example of an ascending-bid auction with affiliated signals of bidders,[3] the literature is mainly silent on the role of affiliation between the signals of the seller and the bidders. This paper's aim is to contribute to the related literature by scrutinizing both the optimal design and the optimal size of reserve prices in such an environment. Our results suggest that affiliation between the signals of the seller and the bidders has important implications for the optimal reserve price: the scope of a secret reserve price is completely undermined, which is in stark contrast to Vincent's (1995) result relying on affiliation among bidders' signals.

3 The model

We consider a two-stage game in which the seller decides on the reserve price design in the first stage, and the auction is run in the second stage. In the first stage, the seller commits to a public reserve price, a revealed reserve price or a secret reserve price. After his decision, he lets the auctioneer know his choice and the reserve price. The auctioneer then either announces the reserve price in case of a public reserve price, or announces that the reserve price will be revealed once it is reached (revealed reserve price), or announces that the reserve price stays secret until the auction is ended by the highest bid. In all cases, the item will be sold only if the highest bid will have reached the reserve price.

In the second stage, the auction is run, and all bidders know the reserve price design. We assume n bidders, and their payoff is normalized to zero if a bidder does not win the auction. If she wins, the payoff of bidder i is equal to $u(\sigma_i, s) = \alpha\sigma_i + (1-\alpha)s$, where $0 < \alpha < 1$. σ_i is the signal received by bidder i, and all bidders draw their $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ from the cdf $F(\sigma)$ that is continuously differentiable, that is, the pdf $f(\sigma) = F'(\sigma)$ exists. s is the seller's signal where $s \in [\underline{s}, \overline{s}]$ is drawn from the cdf $\Psi(s)$ with pdf $\psi(s) = \Psi'(s)$, and α measures the relative importance of the bidder's signal compared to the seller's signal. The seller's payoff of keeping the item is equal to $v(s) = \gamma s, \gamma > 0$, and equal to the price for which the item sells if it sells. We assume that $\underline{\sigma} > \underline{s}(\gamma - (1-\alpha))/\alpha$ holds such that the sale is always mutually beneficial. Furthermore, we make

[3] Vincent (1995) argues that a secret reserve price may increase a seller's expected revenue, especially through the increase in the participation rate led by keeping the reserve price secret

Assumption 1. $\gamma > 1 - \alpha$.

Assumption 1 is a rather weak assumption as it specifies that the seller's signal has a stronger effect on the seller's valuation than on any bidder's valuation. It will guarantee that the reserve price function has the desired properties. We conduct our analysis by scrutinizing an ascending bid auction format and will discuss the implications for other auction formats later. Suppose that a strictly monotonically increasing reserve price function $r^*(s)$ exists, that is, $r^{*\prime}(s) > 0, \forall \in [\underline{s}, \overline{s}]$, a property that we have to show to hold in what follows. In this case, an inverse reserve price function $\theta(r) = r^{*-1}(r)$ exists that is strictly monotonically increasing as well.

Suppose that the seller has set a public reserve price, announced before the auction has started. In this case, given $r^*(s)$, each bidder learns her own valuation and the maximum bid is given by

$$\beta_i(s) = \alpha\sigma_i + (1 - \alpha)s. \tag{1}$$

Now suppose that the seller does not announce the reserve price before the auction begins, but has set a revealed reserve price. Bidders will learn when the bid has reached the reserve price such that the item will sell for this bid or any higher. Before the reserve price is met, the bidding behavior is now given by the maximum bid

$$\beta_i(b) = \alpha\sigma_i + (1 - \alpha)\theta(b). \tag{2}$$

This maximum bid depends on the bid b that is on the floor now: each bidder does not want to suffer from the winner's curse, and she will win only if the reserve price is met, and hence the maximum bid increases with the bid b. A crucial requirement is that $(1 - \alpha)\theta'(b) < 1$ holds. An increase in the bid b by one unit should not increase the maximum bid by more than that. Once the reserve price is met, bidding behavior changes from (2) to (1) as the reserve price and the seller valuation is public now.

Figure 1 visualizes the two bidding strategies for a public and a revealed reserve price. The arrow in both panels shows how the auction proceeds from low to high prices (if not stopped before), starting with the bid $\underline{b} > 0$. In the left panel, we consider a bidder whose bid will not reach the reserve price r. In case of a public reserve price, $\beta_i(s)$ does not depend on b as the bidder knows her valuation right from the start. If the reserve price is only revealed when $b = r$, $\beta_i(b)$ increases with b, but once $\bar{b}$ is reached, the bidder will drop out as his maximum valuation will be below the $b = b$–line for any higher bid. The right panel of Figure 1 has the case of a bidder who will reach the reserve price. In case of a public reserve price, $\beta_i(s)$ is obvious. In case of a revealed reserve price, the bidder learns the seller's signal and her valuation is still above the reserve price. Once b reaches r, there is no difference in the bidding strategies.

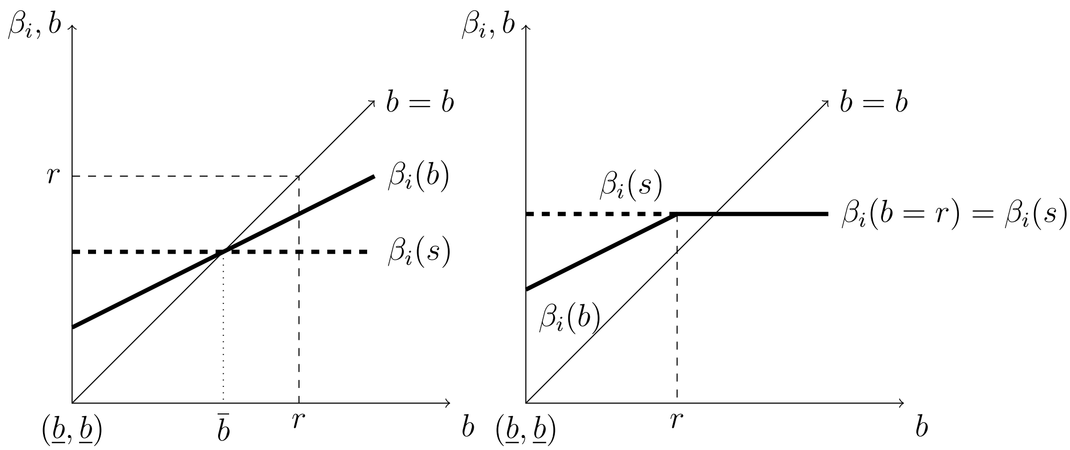

Figure 1: Bidding strategies

What should be immediately clear is that it does not make any difference whether the reserve price is public or revealed. The item will sell only if it is reached, so the difference in bidding behavior before it is reached is inconsequential. Therefore, the optimal reserve price should be the same for a public and a revealed reserve price. In particular, the seller knows that all bidder types for which

$$\sigma \leq \frac{r - (1 - \alpha)s}{\alpha} \equiv \rho$$

holds will not bid up to the reserve price. Routine calculations then show that the expected payment of any bidder that will reach the reserve price is equal to

$$m(\sigma, \rho) = rG(r) + \int_r^\sigma (\alpha\tau + (1 - \alpha)s)\, g(\tau)d\tau \tag{3}$$

where $G(\cdot)/g(\cdot)$ is the distribution of the highest valuation of all other bidders. Furthermore, the expected revenue per bidder is then given by

$$\mathrm{E}\left[m(\sigma, \rho)\right] = rG(\rho)\left[1 - F(\rho)\right] + \int_\rho^{\overline{\sigma}} \left[1 - F(\sigma)\right]\left[\alpha\sigma + (1 - \alpha)s\right] g(\sigma)d\sigma. \tag{4}$$

The seller maximizes his expected payoff

$$V(r) = n\mathrm{E}\left[m(\sigma, \rho)\right] + F(\rho)^n \gamma s \tag{5}$$

w.r.t. r. The expected payoff is equal to the number of bidders times the expected revenue per bidder plus the probability of not selling times the utility of keeping the item. Let $\Phi(\cdot) = G(\cdot)\left[1 - F(\cdot)\right]$, and let $h(\cdot) = f(\cdot)/[1 - F(\cdot)]$ denote the hazard rate for which we assume that $h'(\cdot) > 0$. The first-order condition is

$$\Phi(\rho^*)\left(1 - \frac{r^* - \gamma s}{\alpha} h(\rho^*)\right) = 0 \quad \text{where} \quad \rho^* = \frac{r^* - (1-\alpha)s}{\alpha}.$$

We find:

Proposition 1. *The reserve price function $r(s)$ is strictly monotonically increasing and implies $(1-\alpha)\theta'(b) < 1$ if Assumption 1 holds.*

Proof. See Appendix. □

Note that, as in (optimal) standard auctions, also in this setup n does not play a role for the optimal reserve price. As the reserve price increases monotonically with s, we now also show:

Lemma 1. *The maximized surplus of the seller increases with s.*

Proof. Let $V^*(s) = V(r^*, s)$. The envelope theorem implies

$$\frac{dV^*(s)}{ds} = \frac{\partial V(r^*, s)}{\partial s} = nr^* \frac{1-\alpha}{\alpha} f(\rho^*) G(\rho^*) + \gamma F(r^*)^n > 0.$$

□

Hence, in an ascending bid auction, both the optimal public and the optimal revealed reserve price will increase with s as does the maximized surplus. Sellers who have received a strong signal will not only set a higher reserve price, but the reserve price itself serves as a strong signal for bidders and increases their willingness to pay in the auction. So even if some bidder will decide not to participate in case of a public reserve price as it is below her valuation, this participation effect does not change the optimal reserve price and also not the maximized surplus.[4]

4 Secret reserve prices

What happens if the seller sets a secret reserve price instead? In this case, the bidders do not learn anything during the auction and will have to form an expectation $\hat{s}$ on the seller's type. This expectation $\hat{s}$ will have to replace s in equations (3), (4), (5) and in the first-order conditions. Obviously, if $\hat{s}$ were the unconditional expectation of s, then types with an s less than $\hat{s}$ would win and types with a larger s than $\hat{s}$ would lose. Thus, we may expect that good types will not want to set a secret reserve price, contradicting the assumption that $\hat{s}$ is the average s. We find:

Proposition 2. *No equilibrium exists in an ascending bid auction in which any seller type $s \in]\underline{s}, \overline{s}]$ will employ a secret reserve price.*

Proof. We can do the proof by contradiction. Let the set of all types that select a secret reserve price be denoted by $\mathcal{S}$, and let $\tilde{s} \neq \underline{s}$ denote the type that has the largest realization of s in this set. If all bidders expect that these types select a secret reserve price, $\hat{s} = \mathrm{E}[s \in \mathcal{S}] < \tilde{s}$ must hold according to the Intermediate Value Theorem. Due to Lemma 1, type $\tilde{s}$ is better off by making the reserve price public or revealed, such that $\tilde{s} \notin \mathcal{S}$. This argument then applies to any type $s \neq \underline{s}$ that will select a secret reserve price and will have the largest realization of s after all types with larger realizations will prefer a public or a revealed reserve price. Only type $\underline{s}$ is indifferent, and consequently $\mathcal{S} = \{\underline{s}\}$ or $\mathcal{S} = \emptyset$ in equilibrium. □

Proposition 2 shows that a secret reserve price cannot play any role in an environment in which the signals of bidders are positively affiliated with the seller's signal. The reason is that better types will prefer a public or a revealed reserve price that allows them to realize a larger expected surplus compared to the average surplus they expect in case of a

[4]While $V^*(\cdot)$ increases with n, this change gives the effect of an additional bidder that draws her signal σ from the range $[\underline{s}, \overline{s}]$, and not the effect of a bidder that has drawn a small σ and decides not to participate. As mentioned above, the bidding behavior until the reserve price is met is inconsequential.

secret reserve price. Consequently, each seller, except for type $\underline{s}$, will not employ a secret reserve price in equilibrium.

How do our results extend to first- and second-price sealed bid auctions? First, the case of a revealed reserve price does not exist as a sealed bid auction does not allow for any revelation during the bidding process. Hence, the choice is only between public and secret reserve prices for these formats. Second, as in the standard second-price auction with independent private values, also in this setup it is a weakly dominant strategy for bidders to bid their own value. As no bidder with a value less than the reserve price can make a positive profit, and as the winner with a value equal to or above the reserve price would pay either the reserve price (should the second-highest bid be below the reserve price) or the second-highest bid, in a second-price auction, the expected payment of a bidder is given by equation (3).

Similarly, in a first-price auction, any bidder with a value just equal to the reserve price would bid the reserve price which would win the auction and would bring non-negative surplus had there been no bidder with a value above the reserve price. As the winner with a value above the reserve price should pay the winning bid, the expected payment of any bidder (insofar as the symmetric equilibrium bidding strategy is considered) in a first-price auction is given also by equation (3). It is then straightforward to show that also in a first-price and a second-price auction, the ex-ante expected revenue per bidder is given by equation (4), and that all the three auction formats would lead to the same expected revenue for the seller.[5] This immediately implies that the case of affiliation between each bidder's and the seller's signals is significantly different from the case of affiliation among bidders' signals, as we find:

Corollary 1. *No equilibrium exists in a first-price or second-price auction in which any seller type* $s \in]\underline{s}, \overline{s}]$ *will employ a secret reserve price.*

[5] For a formal proof of revenue equivalence and for further discussions, see Krishna (2002, Chapter 2).

5 Concluding remarks

This paper has delineated the optimal reserve price policy in a sale of an indivisible good by an auction when the valuation of bidders is positively affiliated with the seller's valuation. This is the case when the consumption of the good is mostly a personal matter, and thus bidders only care about their own private signal as well as about the seller's signal, as the seller will know the good's crucial particulars that are important for the buyer. Although this setup is empirically most relevant, as pointed out by Milgrom and Weber (1982), the literature has been silent on such a significant case. Our analysis has shown that the case of affiliation between each bidder's and the seller's signals proves to be different from the case of affiliation among the bidders' signals. In this setup, revenue equivalence holds for public and revealed reserve prices, but there is no room for a secret reserve price.

Appendix: Proof of Proposition 1

Let us write the first-order condition as an implicit function

$$\phi(r^*, s) = \Phi(\rho^*)\left(1 - \frac{r^* - \gamma s}{\alpha} h(\rho^*)\right) = 0.$$

The second-order condition implies that

$$\begin{aligned}
\frac{\partial \phi(r^*, s)}{\partial r} &= \frac{\partial \Phi(\rho^*)}{\partial \rho}\frac{\partial \rho^*}{\partial r}\underbrace{\left(1 - \frac{r^* - \gamma s}{\alpha} h(\rho^*)\right)}_{=0} - \Phi(\rho^*)\left(\frac{h(\rho^*)}{\alpha} + \frac{r^* - \gamma s}{\alpha} h'(\rho^*)\frac{\partial \rho^*}{\partial r}\right) \\
&= -\Phi(\rho^*)\left(\frac{h(\rho^*)}{\alpha} + \frac{r^* - \gamma s}{\alpha} h'(\rho^*)\frac{\partial \rho^*}{\partial r}\right) < 0
\end{aligned}$$

because $h'(\cdot) > 0$ and $\partial\rho^*/\partial r = 1/\alpha > 0$. Since $\partial\rho^*/\partial s = -(1-\alpha)/\alpha < 0$,

$$\begin{aligned}
\frac{\partial \phi(r^*, s)}{\partial s} &= \frac{\partial \Phi(\rho^*)}{\partial \rho}\frac{\partial \rho^*}{\partial s}\underbrace{\left(1 - \frac{r^* - \gamma s}{\alpha} h(\rho^*)\right)}_{=0} + \Phi(\rho^*)\left(\frac{\gamma h(\rho^*)}{\alpha} - \frac{r^* - \gamma s}{\alpha} h'(\rho^*)\frac{\partial \rho^*}{\partial s}\right) \\
&= \Phi(\rho^*)\left(\frac{\gamma h(\rho^*)}{\alpha} - \frac{r^* - \gamma s}{\alpha} h'(\rho^*)\frac{\partial \rho^*}{\partial s}\right) > 0.
\end{aligned}$$

Consequently, $dr^*/ds = -(\partial\phi(r^*, s)/\partial s)/(\partial\phi(r^*, s)/\partial r) > 0$. $(1-\alpha)\theta'(b) < 1$ warrants $dr^*/ds > 1-\alpha$. Let $A \equiv h(\rho^*)/\alpha$ and $B \equiv (r^* - \gamma s)h'(\rho^*)/\alpha^2$, so we can rewrite $dr^*/ds = (\gamma A + (1-\alpha)B)/(A+B)$ which is larger that $1-\alpha$ if $\gamma > 1-\alpha$.

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