Excess Volatility Puzzles

Costas Azariadis
UCLA

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ABSTRACT: Dynamic general equilibrium models of convex economies with complete markets and rational expectations typically predict smooth reactions of household consumption, GNP and asset prices to temporary shocks in household income, total factor productivity and dividends. Actual impulse responses, inferred from cross-sectional and time series data, show considerable volatility.

This essay reviews the evidence on excess volatility and explores, as a potential cause, limits on arbitrage opportunities imposed by solvency constraints and bounded rationality.

1 The issues

- Considerable evidence of high-to-medium frequency fluctuations in (asset prices, GNP, household consumption) relative to fundamental determinants.

- Why would societies that value smooth consumption and input use tolerate "excess" volatility?

- What features/flaws in economic organization permit this form of overreaction to fundamentals?

2 Some empirical puzzles

Puzzle One (asset prices and returns): Share prices vary too much relative to dividends [see Figure 1]

- The standard deviation of the detrended S&P price index is about 30 times the corresponding standard deviation of detrended dividends [Cooley and Quadrini (1998); annual data].

- Innovations in dividends and interest rates are small (e.g., 5-40%) relative to innovations in share prices [Campbell and Kyle (1993)].

Violates: Efficient markets hypothesis [Fama (1964)]

Puzzle Two (negative bubbles): Asset prices fluctuate on *both* sides of reasonably computed "fundamental" values [see Campbell (1996) and Figure 2].

Violates: Arbitrage condition [Tirole (1985)]

$$\begin{aligned} P_t &= PV_t(\text{dividends}) + PV_t(\text{terminal price}) \\ &\equiv \text{fundamental} + \text{bubble} \\ &\geqq \text{fundamental} \end{aligned}$$

Puzzle Three (amplification of TFP shocks):

- Following one-time positive shock, the growth rate of per capita GDP accelerates away from trend for 3-4 quarters [see Figure 3].

Violates: Any reasonably parameterized model of convex economic growth [Solow, Cass-Koopmans, OLG].

Puzzle Four (rate-of-return anomalies):

- Yields of riskless assets (0.7% p.a. for US Treasury Bills) too small when compared with reasonable estimates of time preference and intertemporal elasticity of substitution. [see Figures 4 and 5].

- Yields of stockmarket indices (7.2% p.a. for US for 1947-1993) too high relative to reasonable estimates of risk-aversion parameters [Mehra and Prescott (JME 1985)] or habit formation.

Violates: Unexploited arbitrage opportunity. Risky assets "too cheap" relative to riskless ones.

Puzzle Five (excess sensitivity of consumption to current income):

- Young households behave like buffer stock consumers [see Figure 6].

- Permanent income hypothesis fails in budget studies [Carroll and Summers (1991), Gourinchas and Parker (1999)] as well as in time series studies [Bachetta and Gerlach (1997)].

3 A pair of leads

- Lead One: Volatility due to non-convexities (IRS, fixed costs,...)

- Lead Two: Volatility due to insufficient arbitrage in convex economies (some markets fail; some agents are boundedly rational).

- Evidence in favor of L2 more convincing than evidence in favor of L1. For example:

  – Failure of PIH in household consumption [Carrol and Summers (1991),...,Gourinchas and Parker (1999)].

  – Fit of "noise trading" models in finance [Campbell and Kyle (1993)].

4 A first pass: Multiple equilibrium asset prices

- Volatility and "overreaction" as a motivation for the study of multiple equilibria.

- Key role of "big income effects" for multiple equilibria in OLG pure-exchange economies.

- EXAMPLE:

  – Pure exchange OLG with constant population, stationary endowments and stationary preferences.

  – Assets: consumption loans yielding R_t, "tree" in fixed supply ($= 1$) with dividend $d > 0$. *No* other stores of value.

  – Lifecycle: $L = 2$.

– Community excess demand for consumption goods

$$
\begin{aligned}
D_t &= R_{t-1}s(R_{t-1}) - s(R_t) \\
&\equiv \text{excess demand by old} \\
&\quad + \text{excess demand by young}
\end{aligned}
$$

where

$$s(R_t) = \text{saving by young households}$$

– Markets: competitive and complete (except for claims contingent on "beliefs").

• Equilibrium:

– Market clearing: For $t = 1, 2, ...$

$$D_t = d \qquad \text{or} \qquad s(R_t) = P_t \equiv \text{"tree" price}$$

– Arbitrage: For $t = 1, 2, ...$

$$
\begin{aligned}
R_t &= \frac{P_{t+1} + d}{P_t} \\
\text{ROR on loans} &= \text{ROR on tree}
\end{aligned}
$$

- Properties of equilibrium:
  - sequence $(P_t, R_t)_{t=1}^{\infty}$ may be unique (GS) or indeterminate (not GS) [Figure 7a (7b)].
  - possibility of cycles in asset prices and returns
  - price $= PV$ of dividends for each equilibrium [in general, whenever dividends grow no slower than endowment].
  - positive asset bubbles possible only if tree growth rate $<$ endowment growth rate.
  - negative asset bubbles impossible for positively priced assets (arbitrage).
  - alternating-sign bubbles impossible [Tirole (1985): bubble grows at the rate of interest].

5 Lessons from multiple equilibrium models

- Overreaction to external impulses is consistent with rational economic behavior in convex economies.

- Overreaction is based on weakening of GS: set-valued dynamics.

- Market structure is too rich and forecasting ability is too good to account for anomalies in asset prices and yields.

- Volatility is a property of some, but not all, equilibria [selection problem].

6 The origins of excess volatility

Excess volatility $\equiv$ overreaction to news about anticipated incomes or asset/commodity prices.

Mechanism relies on solvency constraints or bounded rationality:

$$
\begin{aligned}
D_t^h &= \text{demand for commodity/asset by household } h \text{ at } t \\
P_s &= \text{certainty "equivalent" market price at } s \geqq t \\
P_s(h) &= \text{forecast of market price } P_s \text{ by household } h \\
\pi_s &= \text{certainty "equivalent" shadow price at } s \geqq t
\end{aligned}
$$

$$
\pi_s \geqq P_s
$$

with $=$ if household is not demand constrained at s

Then

$$D_t^h = \begin{cases} D(P_t, P_{t+1}, ..., P_T) & \left\{ \begin{array}{l} \text{with rational} \\ \text{expectations and} \\ \text{slack constraints} \end{array} \right. \\ D(\pi_t, \pi_{t+1}, ..., \pi_T) & \left\{ \begin{array}{l} \text{with rational} \\ \text{expectations and} \\ \text{tight constraints} \end{array} \right. \\ D(P_t, P_{t+1}(h), ..., P_T(h)) & \left\{ \begin{array}{l} \text{with bounded} \\ \text{rationality and} \\ \text{slack constraints} \end{array} \right. \end{cases}$$

- Demand reaction to price news: For any $s \in \{t, ...T\}$:

$$\frac{\partial D_t^h}{\partial P_s} = \begin{cases} \sum_{i=0}^{T} D_i \frac{\partial \pi_{t+i}}{\partial Ps} & \text{if tight constraints} \\ \sum_{i=0}^{T} D_i \frac{\partial P_{s+i}(h)}{\partial Ps} & \text{if bounded rationality} \end{cases}$$

where D_i are the partial derivatives of demand.

- Bottom line: shadow prices or forecast prices may be extremely sensitive to market prices.

- Examples:

  1. *Small* change in current wage has strong effect on current consumption/leisure choice of tightly rationed worker. "Shadow" rate of interest (tightness of credit constraints) drops *a lot*.

  2. Small change in *asset price* has strong effect on current demand by boundedly rational household if it is misinterpreted as a *permanent* price change [cf. literature on "robust control"].

7 Solvency constraints

- Loans without repayment commitment.

- Enforcement by Individual Rationality Constraints:

  – borrowers prefer solvency, and subsequent market participation, to default and subsequent exclusion from intertemporal markets.

- Implied solvency constraints reflect:

  – length of market exclusion $\mathcal{L} = 0, 1, ..., \infty$.

  – gains from trade (value of consumption smoothing) during exclusion.

$$
\begin{aligned}
\mathcal{L} &= 0 \text{ [Bewley (1980)] } \Rightarrow \text{ no borrowing} \\
\mathcal{L} &= \infty \text{ [Kehoe and Levine (1993)] } \Rightarrow \begin{cases} \text{generous credit} \\ \text{limits} \end{cases} \\
\mathcal{L} &= 1 \text{ [Azariadis and Lambertini] } \Rightarrow \begin{cases} \text{credit limit and} \\ \text{shadow prices} \\ \text{react strongly to} \\ \text{exclusion period} \\ \text{yield} \end{cases}
\end{aligned}
$$

- Intertemporal substitutability:
  - for households borrowing now and in the future, high values of future yields $(R_{t+1}, ..., R_{t+n-1})$ relax current borrowing constraints and raise planned future consumption.
  - (c_t, c_{t+n}) become complements for constrained agents.
  - Reason: improved ability to service loans after t without cutting much into $(c_{t+1}, ..., c_{t+n-1})$. Higher value of remaining solvent.

7.1 The equity premium and the risk-free rate with $\mathcal{L} = 0$ [Bewley (1980), Krusell and Smith (1998), Huggett (1993)]

- Bewley-type pure exchange economy with heterogeneous infinitely-lived agents $h = 1, ..., H$.

- Deterministic income vector $\{y_t(h)\}_{t=0}^{\infty}$ for $h = 1, ..., H$.

- Stores of value:

  – riskless tree with dividend $d_t^1 = d > 0$ for all $t \geqq 0$

  – risky tree with i.i.d. dividend

$$d_t^2 = \begin{cases} 0 & \text{w.p. } 1 - \pi \\ e > d & \text{w.p. } \pi \end{cases}$$

– Tree prices $\{P_t^1, P_t^2\}$

- Rates of return

$$(R_t^1, R_t^2) = \left(\frac{P_{t+1}^1 + d}{P_t^1}, \frac{P_{t+1}^2 + d_t^2}{P_t^2} \right)$$

- Without solvency constraint

$$\begin{aligned} P_t^1 &= P^1 \qquad \forall t \\ R_t^1 &= 1 + \frac{d}{P^1} = \text{risk free rate} \\ P_t^2 &= P^2 \qquad \forall t \\ R_t^2 &= 1 + \frac{d_{t+1}^2}{P^2} = \text{risky rate} \\ \mathsf{E} R_t^2 &= 1 + \frac{(1-\pi)e}{P^2} \end{aligned}$$

Equity premium

$$\frac{(1-\pi)e}{P^2} - \frac{d}{P^1} > 0$$

reflects pure risk.

- With solvency constraints, $\alpha_t(h, d^2) \geqq 0$
  - all households hold non-negative stock of claims at each $t \geqq 0$ for all realizations of the risky dividend.
  - absence of borrowing means over-saving

$$\hat{P}^1 > P^1 \qquad \hat{P}^2 > P^2$$

  - absence of contingent claims means asset portfolios are *overweight* in safe tree:

$$\begin{aligned} \hat{P}^1 &\gg P^1 \\ \hat{R}_1 &< 1 + \frac{d}{P^1} \\ \mathsf{E}\hat{R}_2 - \hat{R}_1 &> \frac{(1-\pi)e}{P^2} - \frac{d}{P^1} > 0 \end{aligned}$$

(positive equity premium)

7.2 Excess sensitivity with $\mathcal{L} = 1$ [Azariadis-Lambertini (2000)]

- Focus on overreaction. Ignore asset price implications.

- Model:

  - pure exchange OLG with

    * lifecycle = 3

    * identical households

    * one perishable good

    * one store of value in zero net supply (= consumption loans)

  - cohorts $v = -1, 0, 1, ...$

  - time $t = 1, 2, ...$

- endowment:

$$\omega^v = (e_0, e_1, e_2) \qquad v \geqq 1$$

- utility

$$\begin{aligned} V^v &= u(c_v^v) + \beta u(c_{v+1}^v) + \beta^2 u(c_{v+2}^v) \\ v &\geqq 1, \beta > 0 \end{aligned}$$

- Example:

$$\begin{aligned} \omega^v &= (e, 1, e) \qquad 0 \leqq e \leqq 1 \\ u(c) &= \log(c) \\ \beta &= 1 \end{aligned}$$

- Initial condition: obligation of generation $v = 0$ to $v = -1$

$$w_0 \in [0, e_1]$$

- Issues: for a generic initial condition $w_0 \in [0, e_1]$, describe equilibrium when loans are enforced by solvency constraints. How does economy react to wealth/income shocks?

– Answers: Stationary equilibrium [Figure 8]. Three stationary allocations indexed by yields:

$$R_u^* \Rightarrow \text{ slack constraints}$$

$$R_c^* < R_u^* \Rightarrow \text{ binding constraints}$$

$$R \in [0, \bar{R}_1] \Rightarrow \text{loan market closed}$$

– Answers: Dynamic equilibrium [Figure 9].

  * Set-valued perfect-foresight dynamics with sequential indeterminacy (countable infinity of degrees of freedom).

  * Overreaction to unanticipated changes in wealth distribution for open dense sets of initial conditions near $\bar{w}_1$ and $\bar{w}_2$.

8 Bounded rationality

- Mis-specification: Confusion between temporary and permanent shocks (price movements, dividend announcements).

- Who are the boundedly rational agents?
  - inexperienced traders with small information sets
    * “noise” and “liquidity” traders in finance [Kyle]
    * random or imitative demand [Hirshleifer et al]
    * “newly” born in OLG settings
  - rational versus boundedly rational trading
    * forward and backward focus

    - large and small information sets

- A Walrasian framework for bounded rationality
  - auctioneer at t announces price vectors $P^t = (P_s)_{s=t}^{\infty}$
  - rational agents are forward looking
    - express demands conditioned on entire P^t
  - boundedly rational agents are backward looking
    - demands conditioned on price history $(P_s)_{s=t-1}^{0}$, current price P_t, forecasts of future prices, $(P_s^e)_{s=t+1}^{\infty}$
  - auctioneer finds fixed point $(P_s^*)_{s=t}^{\infty}$

- Implications for intertemporal substitutability

- boundedly-rational demands that satisfy gross substitutes at forecast prices may violate GS at actual prices

- An elementary business cycle example [Galor (1996)]
  - standard two-period pure exchange OLG with "identical" agents.
  - each generation contains fraction $1-\lambda \in [0,1]$ of boundedly rational agents who look backwards and fraction λ fully rational agents who look forward:

$$\begin{aligned} \text{Rational households of generation } t &: \\ e_0 - c_t^t &= s(R_t) \\ \text{Bounded household of generation } t &: \\ e_0 - c_t^t &= s(R_{t-1}) \end{aligned}$$

  - equilibrium:

$$\lambda s(R_t) + (1-\lambda) s(R_{t-1}) = 0$$

- conclusions [Figure 10]:
  - given the initial beliefs of cohort $v = 1$, equilibrium is unique. For any $\lambda \in (0, \frac{1}{2})$, yields oscillate en route to the steady state $s(R) = 0$.
  - temporary income shock will be amplified because boundedly rational agents mis-perceive them as permanent shocks.

- Some guesses about asset price implications
  - Arbitrage:
    - no limitations for rational/unconstrained agents
    - prices reflect boundedly-rational demands
  - Bubbles:
    - contribution of boundedly rational dividend forecasts

* possibility of negative bubbles in response to mis-perceived innovations to dividends/yields

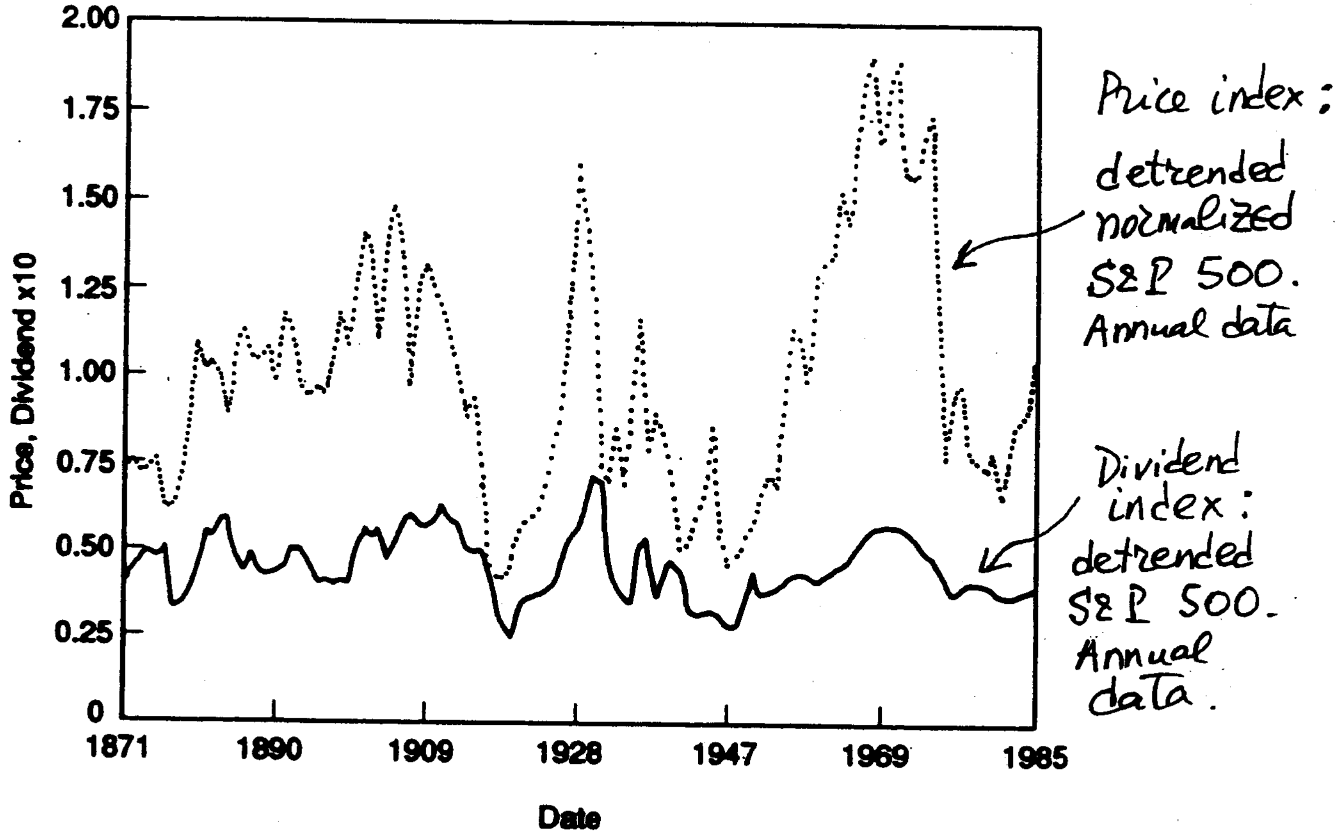
- Question: Are bubbles related to uninformed forecasts following mis-perceived temporary changes in dividends/yields?

9 Conclusions and more questions

- Bounded rationality appears to be essential for understanding behavior of observed asset price bubbles over time. Full model?

- Debt constraints (a sophisticated form of market incompleteness) are useful in rationalizing the persistence of unexploited arbitrage opportunities [see Mehra et al (1997)].

- Overreaction may be understood by studying how "news" changes *forecasts of shadow prices*, rather than realizations of market prices.



Stock price and dividend data

Figure 1.

[Campbell and Kyle, RES 1993].

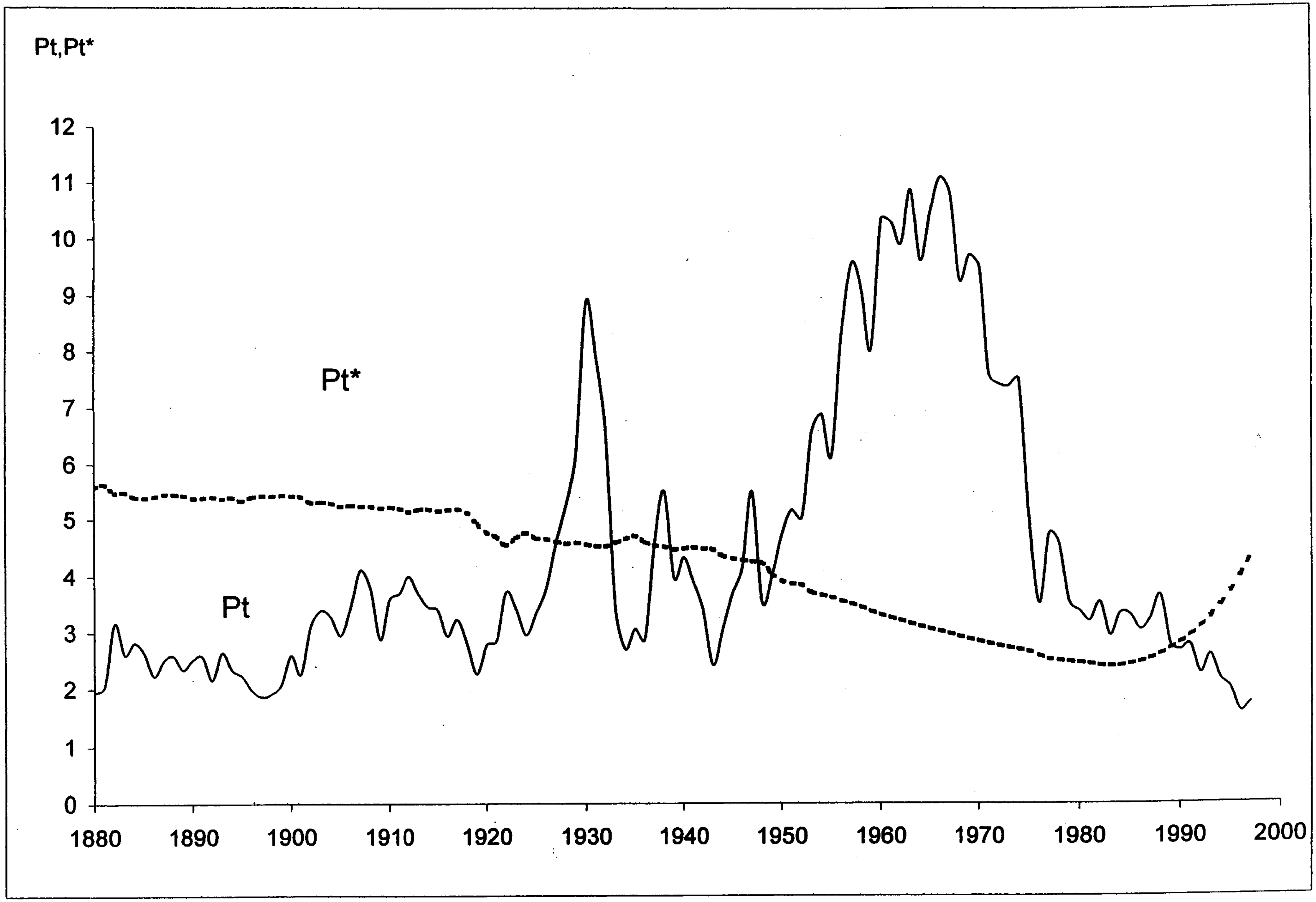


Figure 2.

[Source: data from Shiller's webpage].

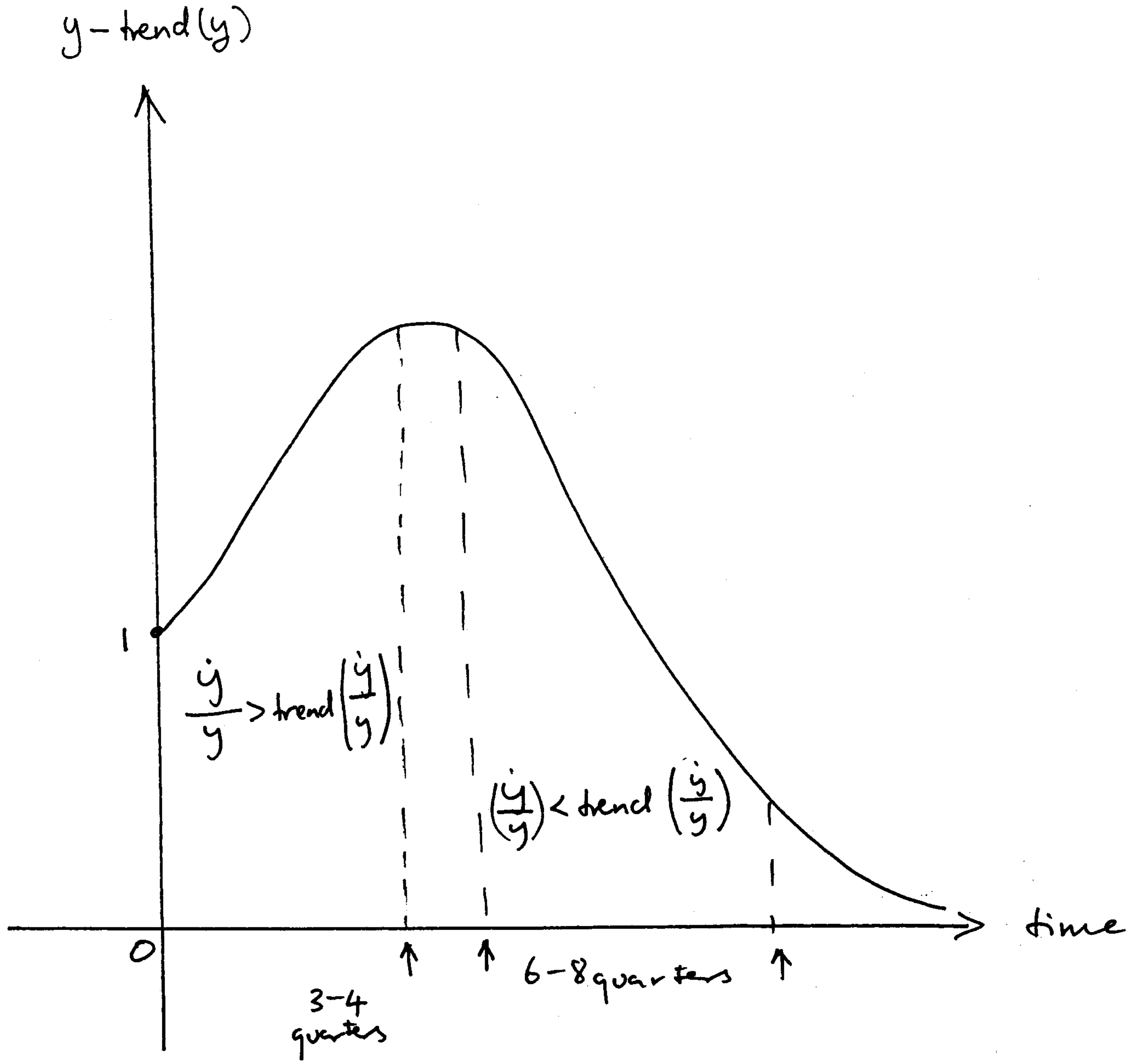


Figure 3.

[Cogley and Nason, 1995].

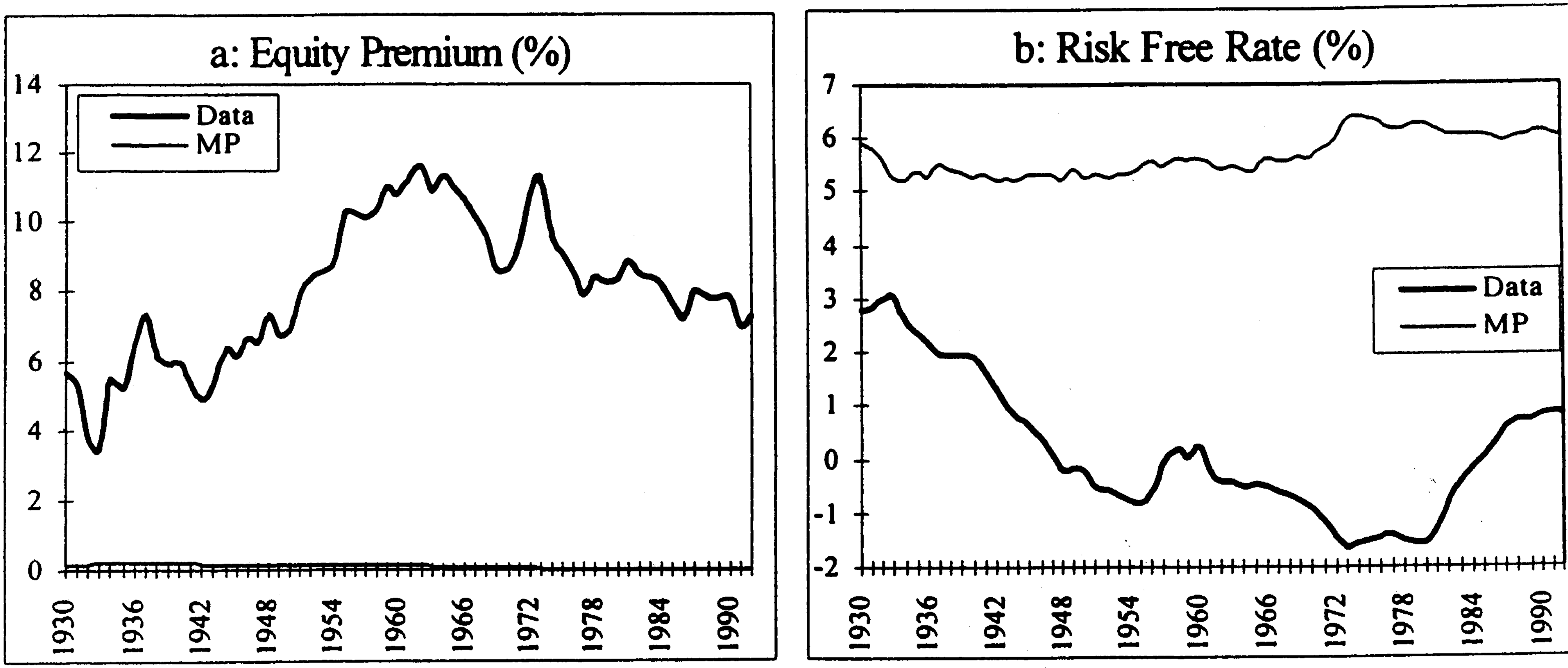


Figure 4.

[Otrok et al (2000)]

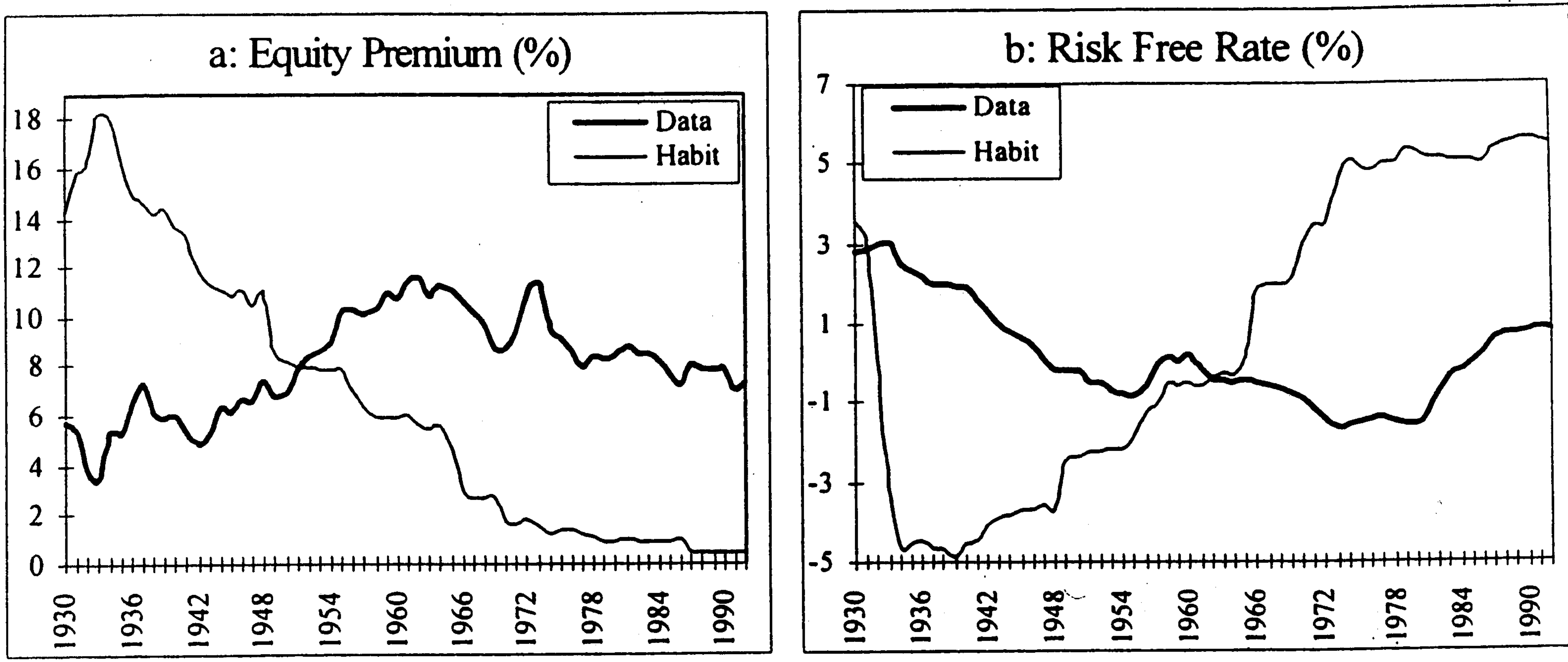


Figure 5.

[Otrok et al (2000)]

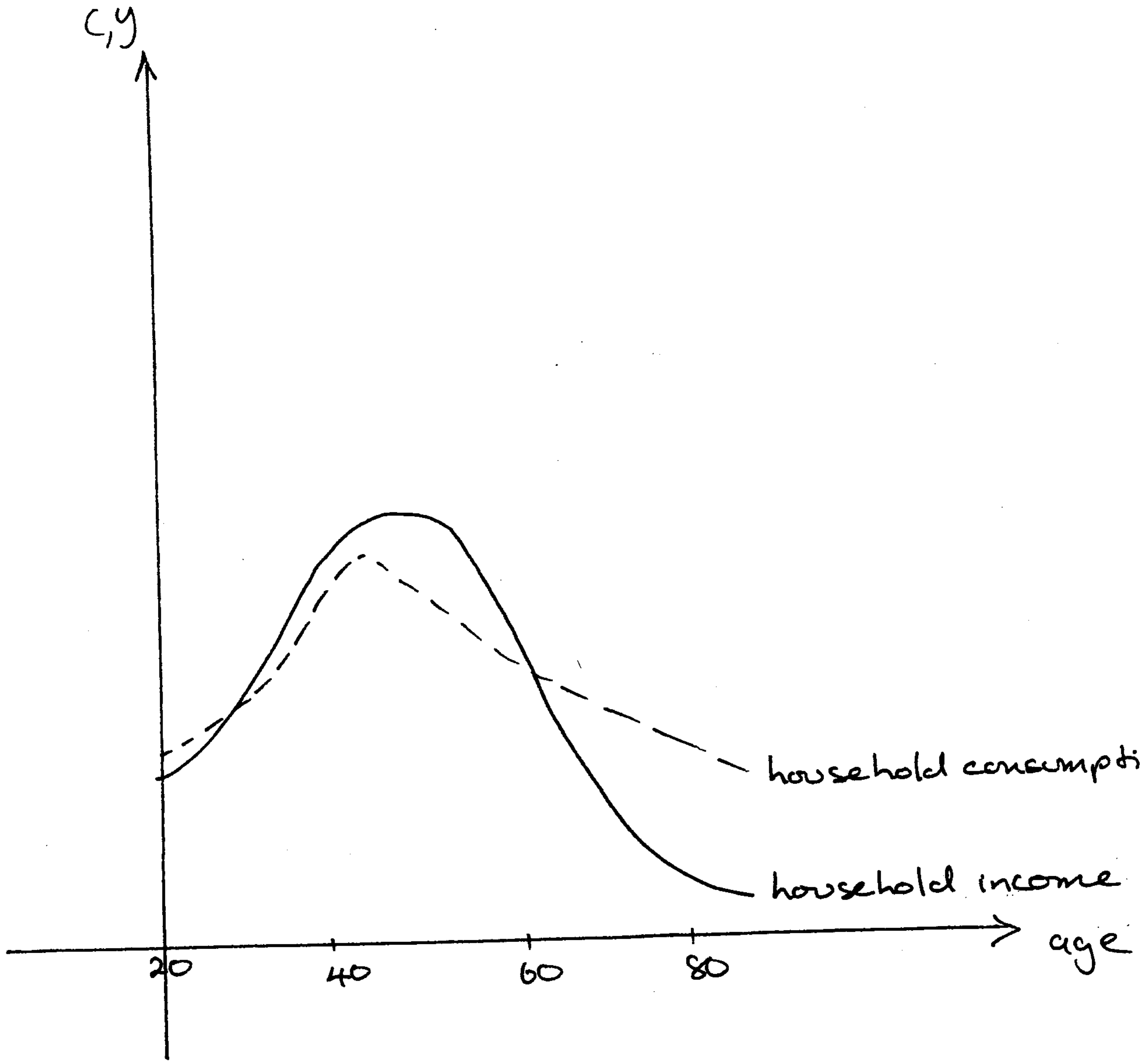
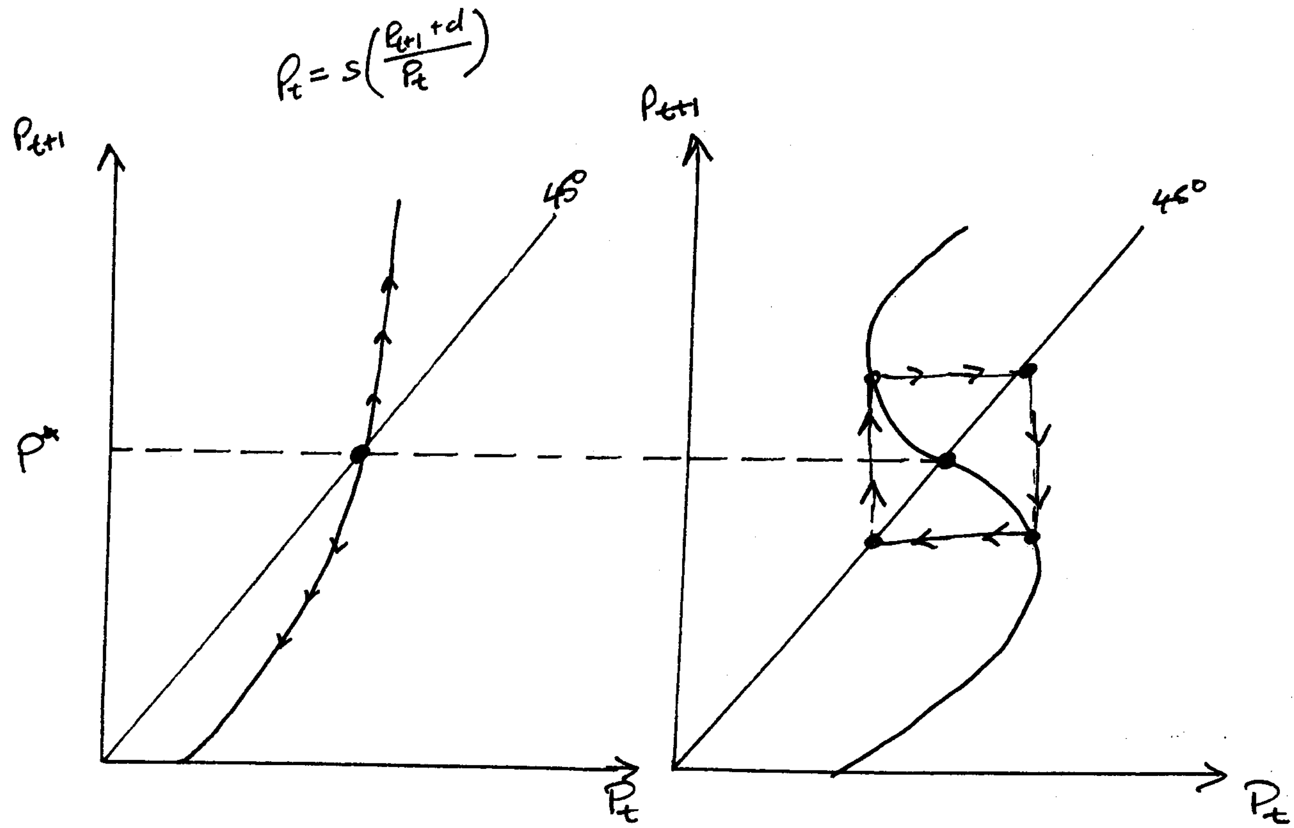


Figure 6.



Gross substitutes — No gross substitutes

Figure 7(a). — Figure 7(b).

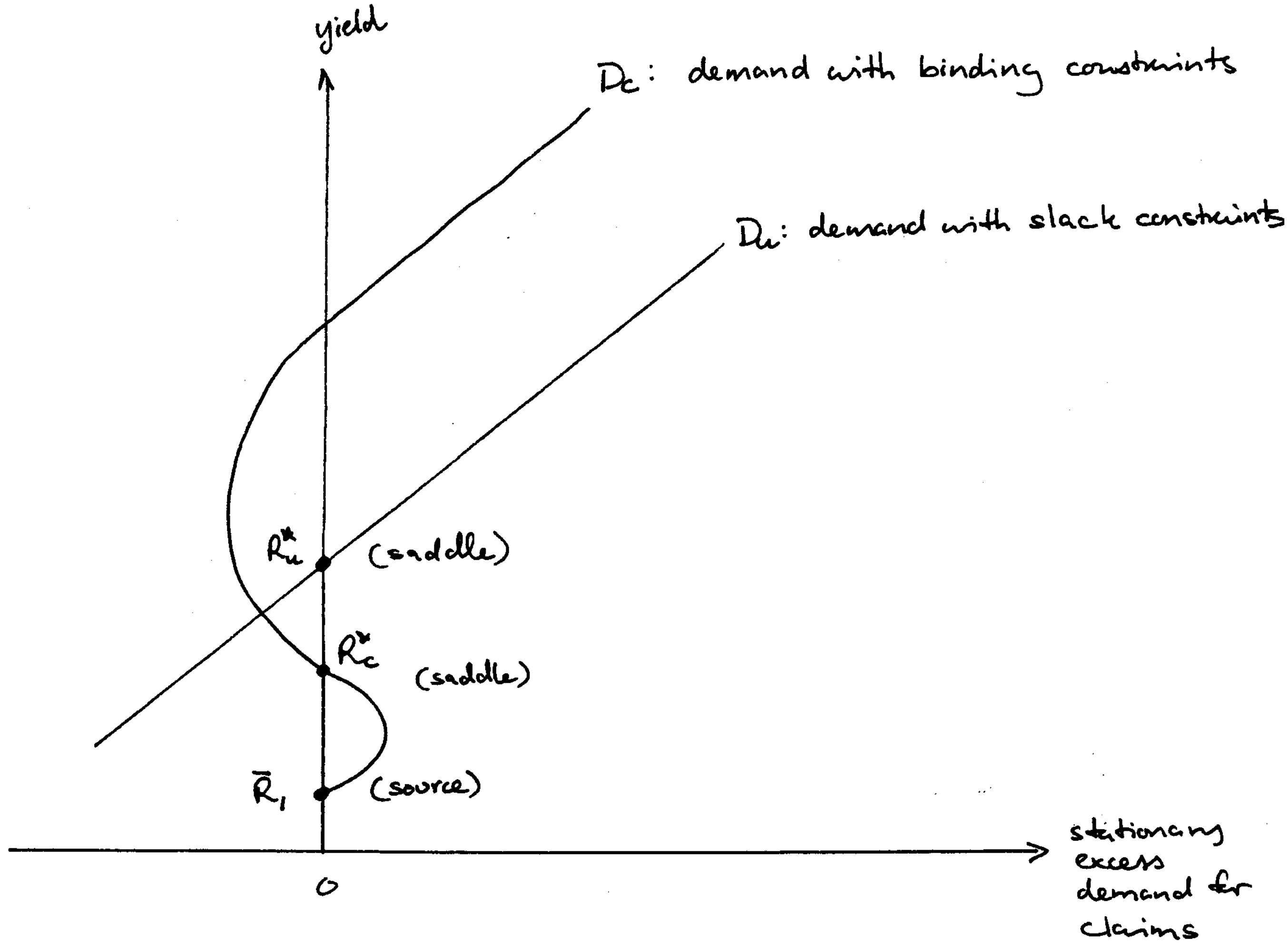

yield
Dc: demand with binding constraints
Du: demand with slack constraints
R*u
(saddle)
R*c
(saddle)
R̄1
(source)
0
stationary excess demand for claims

Figure 8.

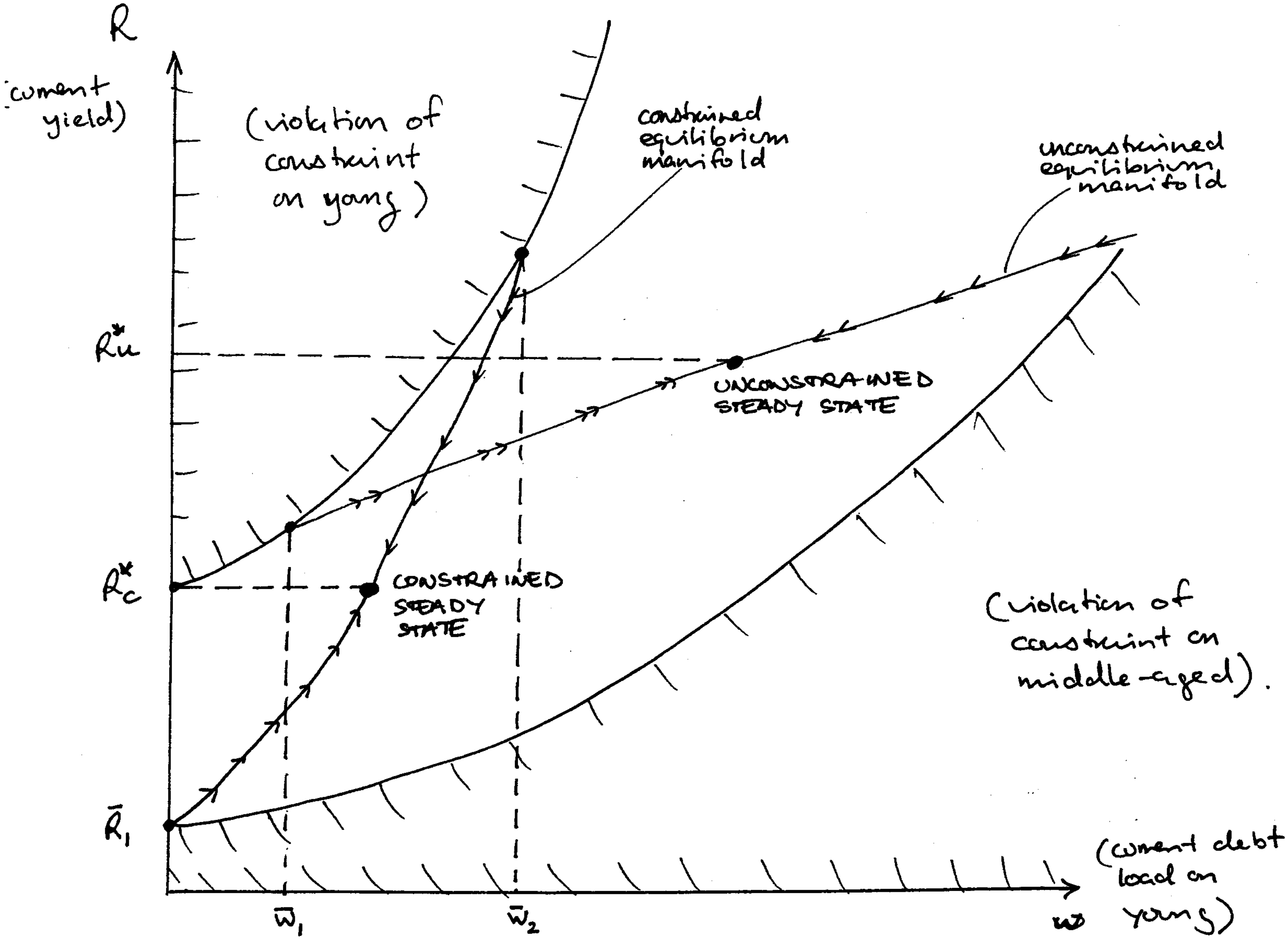

R
(current yield)
(violation of constraint on young)
constrained equilibrium manifold
unconstrained equilibrium manifold
R_u^*
UNCONSTRAINED STEADY STATE
R_c^*
CONSTRAINED STEADY STATE
(violation of constraint on middle-aged).
$\bar{R}_1$
$\bar{w}_1$
$\bar{w}_2$
w
(current debt load on young)

Figure 9.

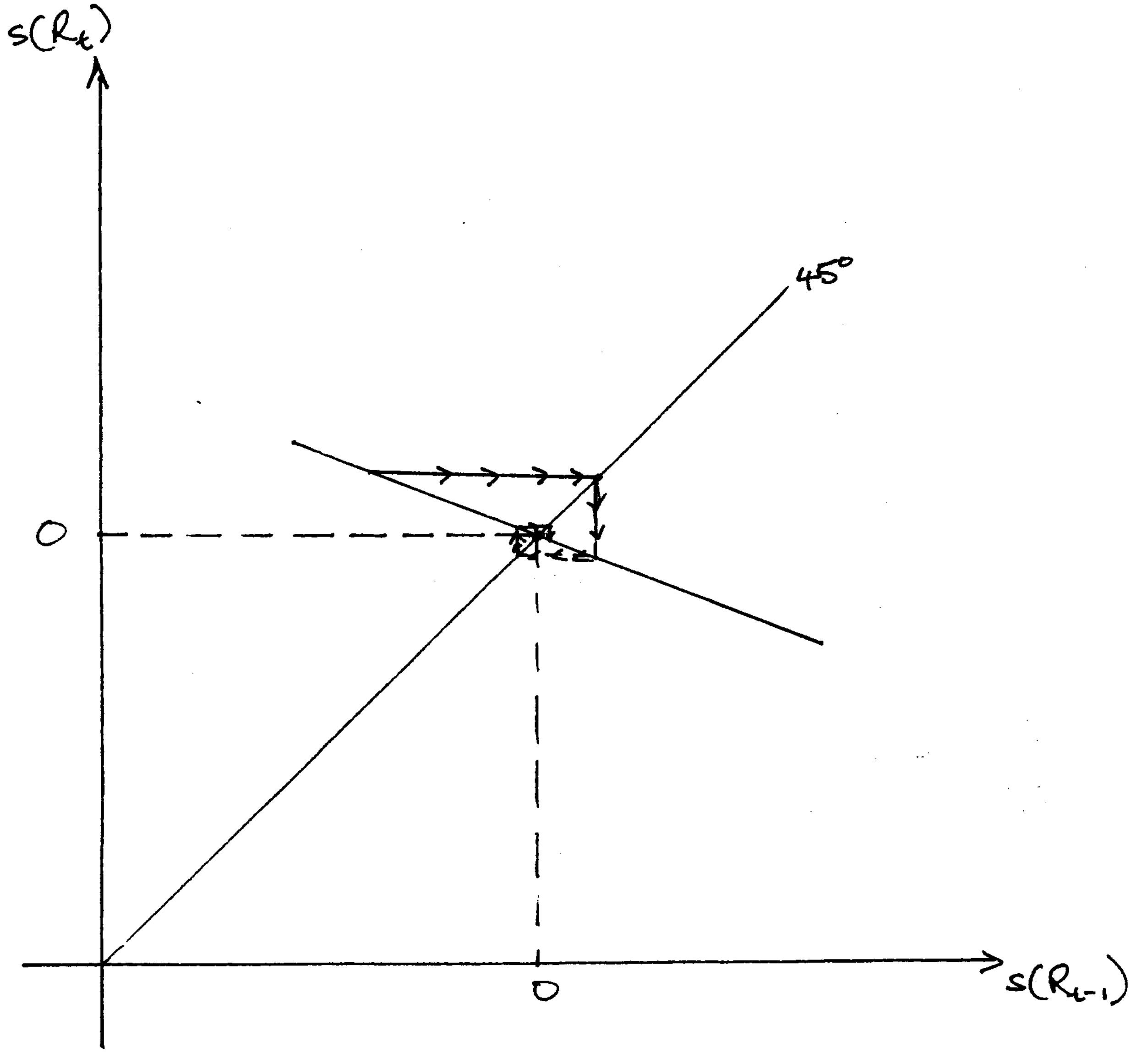


Figure 10.