Growth Maximizing Patent Life-Time

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Abstract

Using a vetical differentiation model of endogenous growth with stochastic R&D activity, we characterize the optimal patent life-time, the government would set in order to maximize economic growth. We show that a finite patent life-time does exist and is unique provided that the expected rate of return from R&D is sufficiently large. Additionally, we analyze the impact of the level of competition in R&D sector, the interest rate, the level of monopoly profit and the productivity parameter of research technology on this optimal patent life-time.

Keywords: Endogenous growth; Patents life-time; Innovation

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1 Introduction

Since Schumpeter (1942), the need to provide innovators with some form of market power (i.e. patent protection) in order to stimulate investment in R&D has been widely recognized. As the patent protection helps innovators to benefit from their research efforts, it creates or increases the incentives to innovate. It also seeks to avoid duplication of research efforts and to promote technological progress.

The analysis of optimal patent life-time is founded on two branches of literature.

In the indutrial organization literature, the analysis is based on the hypothesis that a society must balance the gains accruing from rapid technological progress against the welfare loss associated with the presence of temporary monopolies in the use of new technology. Nordhaus (1969), Loury (1979), and recently Denicolo' (1999), among others, all study the incentives and the distortions induced by a patent system within a static, partial equilibrium framework. Judd (1985) is the first attempt to study the optimal patent life-time through maximizing social welfare function in a dynamic, general equilibrium set-up. He builds an exogenous growth model where innovation, though endogenous, is not sustainable when there is no exogenous increase in the labour force.

Within endogenous growth literature, innovation-based economic growth has become an important field of research due to the works of Segerstrom et al. (1990), Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Barro and Sala-i-Martin (1995). Considering innovations as to create a market for differentiated products by increasing the degree of product variety or quality, in these models no attenction is paid to the patent system as a government policy tool. Indeed, Romer (1990) and Grossman and Helpman (1991, chapter 3) assume that a successful innovator accrues an infinite patent protection. In a recent work, Michel and Nyssen (1998) analyze patent life-time in an endogenous growth model with horizantal product differentiation. However, their analysis is limited in the sense that they consider a deterministic innovative activity. Since Loury (1979), the limits of modelling innovative activity as a deterministic process, instead of a stochastic one, have been widely recognized.

Given all this, it is surprising that the impact of patent life-time as a government policy tool on economic growth is almost entirely absent from theoretical work. Our motivation in the present paper is to fill this gap in the literature. In this sense, we focus on the optimal patent life-time the government would set in order to maximize economic growth in an endogenous technological change model with vertical differentiation and stochastic R&D activity. Within this framework we also study how this optimal patent life-time would change in response to changes in the level of competition in R&D, the interest rate, the productivity parameter of research technology and the monopoly profit.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces economic growth. Section 4 characterizes, through numerical analysis, the growth maximizing patent life-time and performs comparative statics. Finally, Section 5 concludes.

2 The Model

We consider an economy composed of three sectors. Final output sector produces a homogenous consumption good, intermediate sector produces capital goods whereas research and development sector produces innovations. Final output is obtained by combining labour and capital goods through a Cobb-Douglas technology and is used as an input in the intermediate goods and R&D sectors. Firms which operate in the research and development sector race to produce innovations in terms of quality improvements in the existing capital goods' product lines. The successful innovator wins a patent which is licensed to capital goods' producers.

2.1 Final Output Sector

Final Output (the numeraire) is produced in a competitive industry according to the following constant returns to scale production function:

$$Y_t = L^{1-\alpha} \int_0^1 (\tilde{X}_{jt})^{\alpha} dj; \quad \tilde{X}_{jt} \equiv \sum_{m=0}^{M_j} (q^m X_{m_j;t}); \quad 0 < \alpha < 1; \ q > 1 \qquad (1)$$

where $Y; L$ and q denote output, fixed supply labour, and a quality-ladder index respectively. The degree of horizantal differentiation is fixed over time, $j \in [0;1]$. $X_{m_j;t}$ is the quantity used of the j^{th} type of intermediate good with quality level m_j: M_j being the highest quality level available in sector j; $\tilde{X}_{jt}$ is the quality adjusted amount of the variety j employed at time t.

Supposing that for each variety j, only the top quality generation (q^{M_j}) is produced in equilibrium, final output production function can be recast as follows:

$$Y_t = L^{1-\alpha} \int_0^1 (q^{M_j} X_{M_j;t})^{\alpha} dj \qquad (2)$$

The representative firm producing the homogenous final output maximizes its instantaneous profit with respect to X_{Mj}:

$$\max Y - wL - \int_0^1 (P_{Mj} X_{Mj})dj \tag{3}$$

In equation (3), the price of the final good has been normalized to unity and for ease of notation the subscript t has been omitted. The solution to this problem yields the set of intermediate input demand schedules:

$$X_{Mj} = \left(\frac{\alpha q^{\alpha M_j}}{P_{Mj}}\right)^{\frac{1}{1-\alpha}} L \tag{4}$$

The implied price-demand elasticity (ε) faced by each intermediate monopolist is given by $\frac{1}{1-\alpha}$:

2.2 Intermediate Goods Sector

At any point in time, there is a continuum of firms indexed by $j \in [0;1]$ operating in this sector. Each firm produces a differentiated intermediate good that is used in the production of final output as an input. One unit of foregone consumption allows each firm to produce one unit of intermediate good irrespective of its variety. Accordingly, the marginal cost of each firm is equal to one. In order to obtain the optimal price of the highest available quality of each intermediate good, firms equate their marginal cost to their marginal revenue leading to $P_{Mj} = \frac{1}{\alpha}$. Thus, price is a fixed mark-up on the marginal cost of production. A firm that has already incurred the fixed-cost investment in a patent will obtain the following instantaneous profit[1]:

$$\pi_{Mj} = (P_{Mj} - 1)X_{Mj} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} q^{\frac{\alpha}{1-\alpha}M_j} L \;, \quad \forall j \in [0;1] \tag{5}$$

[1] At any point in time, the only sunk cost faced by each firm operating in the intermediate goods sector is the initial expenditure on the patent. This assumption is harmless in the present context as the intermediate input demand is stationary in equilibrium.

2.3 Research and Development Sector

In this section, we postulate an R&D technology where the date of innovation is uncertain. Following Loury (1979), Dasgupta and Stiglitz (1980) and Denicolo' (1999), we assume a poisson process for innovations which will bring discrete quality improvements for the intermediate goods. The timing of innovations stochastically depends on the R&D investment, namely, the amount of final output devoted to R&D. A firm introducing an innovation will maintain a legal patent whose life-time is set optimally by government in order to maximize economic growth.

Concerning the structure of the sector, at time t, each firm determines the amount of resources devoted to R&D through maximizing its expected profit:

$$E[\pi] = \lambda N_i E[V] - N_i - F \tag{6}$$

where

$$E[V] = \Pi_{Mj} \lambda \left(\sum_i N_i \right) \int_0^T e^{-(r+\lambda \sum_i N_i)\tau} d\tau + \frac{\Pi_{Mj}}{r}\left(1 - e^{-rT}\right) e^{-(r+\lambda T \sum_i N_i)} \tag{7}$$

In equation (6), λN_i is the poisson arrival rate of innovation for firm i devoting N_i amount of resources to R&D with $\lambda > 0$ being a parameter reflecting the productivity of the research technology. Apart from foregone consumption, there is a fixed cost F; that firms pay in order to engage in R&D. At any point in time, firm i innovates with probability λN_i and accrues the value of the patent, V:

A firm that obtains a certain quality improvement keeps the private property of this innovation only during the legal patent life-time, T: However, if the next innovation arrives before T elapses, then the actual life-time of the patent will be limited by the duration between these two consecutive innovations. As a poisson process is assumed for innovations, the duration between two consecutive innovations follows an exponential distribution with an arrival rate of $(\lambda \sum_i N_i)$. Accordingly, the expected value of a patent given in equation (7) is composed of two terms that capture the probability of having the next innovation before or after T elapses, with r denoting the interest rate.

Concentrating on symmetric equilibrium, we analyze the case where all firms devote the same amount of resources to R&D in order to improve the quality of the variety which would induce the highest amount of monopoly profit. Thus, without loss of generality, assuming that $N_i = N$ for all i and $\max\{\Pi_{Mj}; j \in [0;1]\} = \Pi$; the first order condition for a maximum becomes:

$$\frac{\beta\lambda^2 N S}{r+\lambda N S}\left(1-e^{-(r+\lambda N S)T}\right)+\frac{\beta\lambda\left(1-e^{-rT}\right)e^{-\lambda N S T}}{r}+\frac{\beta\lambda^2 N r}{(r+\lambda N S)^2}\left(1-e^{-(r+\lambda N S)T}\right)$$
$$+\frac{\beta\lambda^3 N^2 T S e^{-(r+\lambda N S)T}}{r+\lambda N S}-\frac{\beta\lambda^2 N T\left(1-e^{-rT}\right)e^{-\lambda N S T}}{r}=1 \qquad (8)$$

where S is the number of symmetric firms engaged in $R\&D$. It is evident that S is bounded as it is a decreasing function of the fixed $R\&D$ cost, F:

3 Economic Growth

Using (2) and (4), the level of aggregate output (Y) can be recast as:

$$Y = L^{1-\alpha}\int_0^1 \left[q^{M_j}\left(\alpha^2 q^{\alpha M_j}\right)^{\frac{1}{1-\alpha}} L\right]^{\alpha} dj \Rightarrow Y = L\alpha^{\frac{2\alpha}{1-\alpha}} Q \qquad (9)$$

where $Q \equiv \int_0^1 q^{\frac{\alpha}{1-\alpha}M_j} dj$ is the aggregate quality index. Hence, the growth rate of output is equal to the growth rate of Q. For a variety j, the proportionate change in Q due to a successful innovation is $(q^{\frac{\alpha}{1-\alpha}}-1)$ and, under the symmetric equilibrium hypothesis, the flow propobability of a success due to poisson process is $\lambda N S$. Accordingly, the expected growth rate of the economy can be stated as:

$$g = E\left[\frac{\dot{Y}}{Y}\right] = \lambda N S\left(q^{\frac{\alpha}{1-\alpha}}-1\right) \qquad (10)$$

For having a positive economic growth, the amount of resources devoted to $R\&D$ by each firm should be positive as the number of firms engaged in $R\&D$ is bounded. Checking the first order condition given in (8) when $S \to 0$ and $S \to \infty$ leads to the conditions $\lambda\beta > r$ and $\lambda\beta = 1$; respectively, in order to have $N > 0$: $\lambda\beta$ can be interpreted as the expected rate of return from investing one unit of foregone consumption in $R\&D$. Clearly, it should be bigger than the interest rate in order to give sufficient incentive to firms to innovate.

3.1 Growth-Maximizing Patent Life-time

Under symmetric equilibrium, for given S; the government's problem to set the legal patent life-time that maximizes economic growth reduces to maximizing N

with respect to T. Implicit differentiation of (8) yields the following condition for a growth-maximizing patent life-time:

$$\frac{\partial N}{\partial T} = 0 \Rightarrow$$

$$(1 - \lambda N T)(1 + \lambda N S)e^{-(r+\lambda N S)T} + \frac{\lambda N(\lambda N S T - S - 1)(1 - e^{-rT})e^{-\lambda N S T}}{r} + \frac{\lambda N r e^{-(r+\lambda N S)T}}{r + \lambda N S} = 0 \qquad (11)$$

The solution to the government's problem can be obtained through solving (8) and (11) simultaneously. Unfortunately, we have not been able to conduct a complete algebraic analysis and so we have proceeded with numerical simulations. These simulations show that, under the condition $r < \lambda\Pi < 1$, a finite growth maximizing patent life-time does exist and is unique.

4 Numerical Analysis

The aim of our numerical analysis is to characterize the legal patent life-time that maximizes economic growth and to see how it changes with respect to the level of competition in R&D, namely the number of firms engaged in research, interest rate, monopoly profit and the research productivity parameter.

Figure 1 can be used to illustrate the legal patent life-time, T^*, that will be set by government in order to maximize economic growth and the corresponding amount of resources devoted to R&D by each firm, N^*, under the parameter values reported in Table 1[2].

Table 1

Parameter Values and Corresponding T^* and N^*

$\lambda = 0.0005$	$\Pi = 500$	$r = 0.05$	$S = 100$	$T^* = 10.354$	$N^* = 1.575$

In figure 1, according to the conditions given in (8) and (11), we have plotted two curves (with a continous and a dashed line, respectively) that give the amount

[2] We have assigned the value of λ in the line of the emprical findings of Duguet and Kabla (1998). For sake of expositional simplicity, we have assigned arbitrary values for Π, being aware of the fact that, indeed, it depends on the parameters, α, q and M_j. Finally, we have set a usual value for the interest rate.

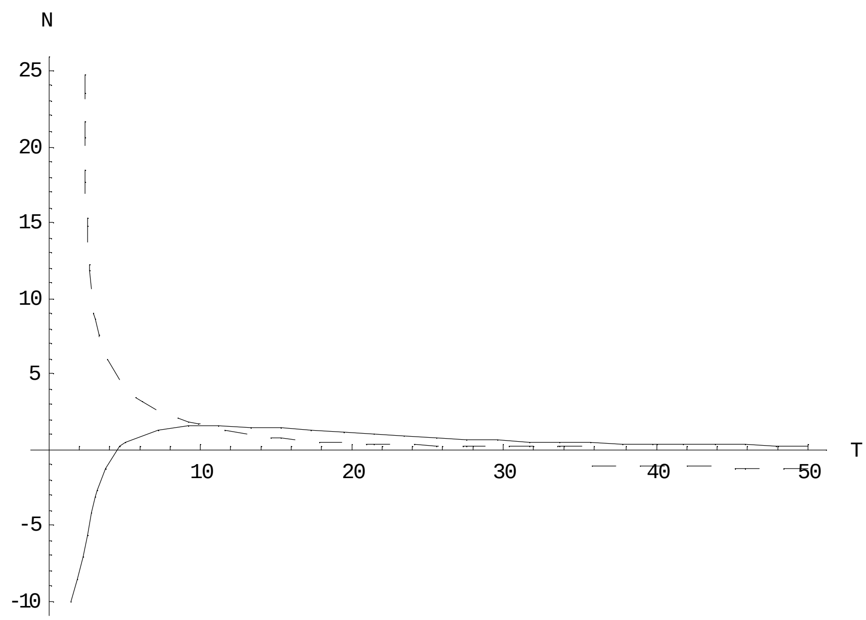

Figure 1: Characterizing Growth Maximizing Patent Life-Time

of resources devoted to R&D by each firm in terms of legal patent life-time. The intersection of the two gives the solution to the government's problem, $T^{\star}$. As is clear from the Figure, for admissible parametirizations, a finite growth maximizing patent life-time exists and is unique.

We observe that an increase in the number of firms, thus the level of competition in R&D sector, leads to a decrease in the number of resources devoted to research by each firm. However it increases the total R&D investment, N¤S.[3] This implies a decline in the expected duration that would elapse between two consecutive innovations, which in turn reduces the incentive of each firm to engage in R&D. To compensate for this, the government sets a higher legal patent life-time in order to maximize N, hence economic growth. In Table 2, for given parameter values (¸; ¦ and r); we have reported $T^{\star}$ and the corresponding $N^{\star}$ values with respect to different numbers of firms in R&D.

[3] Loury (1979) has also confirmed the same relationship between the number of firms, the total amount of R&D investment and the expected time to invention. However, he has not dealt with economic growth and thus growth-maximizing patent life-time, which is the main concern of this paper.

Table 2

The Impact of the Level of Competition in R&D

$\lambda = 0.0005$; $\pi = 500$; $r = 0.05$	$S = 1$	$S = 10$	$S = 100$	$S = 1000$	$S = 10^6$
T^*	9.394	10.297	10.354	10.357	10.358
N^*	71.128	14.412	1.575	0.159	0.016

We also perform numerical exercises for analyzing the impact of different r, λ and π on T^*. The results are contained in Tables 3, 4 and 5.

Table 3

The Impact of the Interest Rate

$\lambda = 0.0005$; $\pi = 500$; $S = 100$	$r = 0.05$	$r = 0.10$	$r = 0.15$	$r = 0.20$
T^*	10.354	10.921	11.983	14.238
N^*	1.575	1.053	0.608	0.247

Table 4

The Impact of Research Technology Parameter

$r = 0.05$; $\pi = 500$; $S = 100$	$\lambda = 0.0002$	$\lambda = 0.0005$	$\lambda = 0.0010$	$\lambda = 0.0015$
T^*	29.175	10.354	4.669	2.709
N^*	0.761	1.575	2.320	3.455

Table 5

The Impact of Monopoly Profit

$\lambda = 0.0005$; $S = 100$; $r = 0.05$	$\pi = 200$	$\pi = 500$	$\pi = 1000$	$\pi = 1500$
T^*	29.175	10.354	4.669	2.709
N^*	0.305	1.575	4.640	10.365

A higher rate of interest implies a lower expected value for a patent as firms would discount future profits more. The incentive to innovate reduces due to a decrease in the expected net gain per unit of resources $(\lambda\pi - r)$ devoted to R&D. Thus, the optimal legal patent life-time maximizing economic growth increases. On the other hand, an increase in the expected net gain per unit of resources (due to an increase in either λ or π) rises the incentive to innovate so that a smaller value of a legal patent life-time would suffice to reach the maximum level of economic growth.

5 Conclusion

In this paper, we have characterized the patent life-time that would be set by government (T^*) in order to maximize economic growth within an endogenous growth model with vertical differentiation and stochastic R&D activity. Through numerical simulation, we have shown that a finite growth maximizing patent life-time does exist and is unique. We have also shown that, T^* increases with the level of competition in R&D sector and the interest rate wheras it decreases with the monopoly profit and the productivity parameter of research technology. For future work, along with patent life-time, the impact of other policy instruments (such as patent breadth) on economic growth would be analyzed.

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