Adoption Costs, Age of Capital and Technological Substitution[1]

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Abstract: In this paper we introduce adoption costs in a vintage capital model. We assume that the incorporation of technological innovations into the production sector requires an extra labor cost during a fixed period. First, we show how adoption crucially matters in the shape of short run and asymptotic dynamics. Then, we analyze the consequences of adoption costs in technological substitution extending the model in two ways: we let adoption costs depend on the technical growth rate, and we endogenize them, depending on the technological gap. When adoption costs depend on the technical growth rate, the effect of growth on optimal lifetime of machines is indeterminate; the creative destruction effect can be compensated by the adoption effect, and faster growth rates delay the technological substitution. Finally, when adoption costs are endogenous, we recover the typical obsolescence effect in vintage capital models and show that technological progress has a negative effect on the technological gap.

Keywords: Machine replacement, Technology adoption, Optimal scrapping, Economic fluctuations..

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1 Introduction

The key assumption in vintage capital technology is that technical progress is embodied in new machines, and new machines are always better than old ones; because of that, it is an appropriate framework to study the adoption of new technologies. A number of papers have stressed the importance of adoption costs in the process of implementing new technologies: Parente and Prescott (1994) assume that adoption is a costly activity and firms' decisions to adopt better technologies imply a trade-off between the additional adoption costs and the increase in the firms' revenues due to the use of a better technology; in their model, adoption costs are exogenous and take the form of institutional and external constraints. Greenwood and Jovanovic (1998) emphasize the effect of adoption costs and their interaction with skill in adopting better technologies. Some empirical papers reinforce the previous idea. Flug and Hercowitz (1997) find that an increase in equipment investment leads to a rise in the demand of skilled labor. Adler and Clark (1991) show that the opening of a plant is followed by a temporary increase in the use of skilled labor whose job is to get the production process "up to speed". In other words, skilled labor facilitates the adoption process.

From a theoretical viewpoint there are two ways to deal with adoption costs. The adoption of a new technology may carry a large forgone output cost incurred during the learning period, as it takes some time for the new technology to operate at its theoretical level (Parente, 1994; Boucekkine and Martinez, 1999). On the other hand, an increase in the labor devoted to adoption, or an increase in production costs, during a temporary period can help the new technology to reach its potential productivity. Magnac and Verdier (1993) analyze in a duopolist framework the process of adoption when the new technology is characterized by adaptation costs and learning effects. They consider that adoption shows two phases. In the first phase, the firm must incur larger costs since it must adapt to the new technology it is using for the first time; in the second period, the firm has already learned how to use the new technology and it is able to produce at lower cost.

In fact, the previous theories are equivalent in that they cause a decline in the firms' net revenues; in the first case this is introduced directly in the model, while in the last case it is assumed that the decrease in productivity during the adoption period is compensated by hiring an extra amount of labor, or extra production costs.

In this paper, we consider the second approach. We study a Ramsey growth vintage capital model with linear utility and Leontief technology, previously studied by Boucekkine, Germain and Licandro (1997), and relax the zero adoption cost they assume. We model adoption costs, following the Magnac and Verdier setup of two adoption phases, as an additional labor cost to be paid over a fixed interval of time. To keep things as simple as possible, we do not impose any skill condition on the labor input required for adoption so that the labor market is not segmented in our model. We show however that our results are not qualitatively altered if a skill differential were to be introduced, especially with respect to the dynamics. Indeed, our main results come essentially from the fact that our specification of adoption involves an adoption period whose duration is significantly different from the time frequency of replacement investment as endogenously determined in the model. What is crucial in our setting is not really the skill of the employees working on the adoption side of the economy but rather the fact that the duration of their specific activity is generally much lower than the optimal lifetime of the production units.

The objective of the paper is to study how the short and the long run dynamics are affected by adoption costs and to analyze the consequences of adoption costs in technological substitution. In contrast to the zero adoption model considered by Boucekkine, Germain and Licandro (1997), in which the dynamics of investment is purely periodic in the short and in the long run, the inclusion of adoption costs gives rise to some "irregular patterns" in the short run and convergence to the steady state in the long run. To deal with the effect of adoption costs on technological substitution we make two extensions of the model. First, we let adoption costs depend on the rate of technical progress in order to analyze if adoption costs destroy the inherent obsolescence effect obtained in vintage capital models. We show that when adoption costs depend on the technical growth rate, the effect of growth on the optimal lifetime of machines is indeterminate. The creative destruction effect can be compensated by the adoption effect, and faster growth rates imply the use of older technologies by firms or countries with higher adoption costs. Finally, we relax the assumption according to which it is always profitable to adopt the best technology, and assume that adoption costs depend on the adopted technology. In this framework we also check the robustness of the obsolescence effect and the influence of the technical progress rate on the technological gap.

The paper is organized as follows. Section 2 describes and shows the centralized equilibrium of the simplest model: adoption costs are constant and exogenous. Section 3 characterizes the steady state of this economy and gives the comparative statics. Section 5 develops the dynamics, in particular, we study the asymptotic stability of the model and the short run behavior. Section 6 and 7 extend the model to check the robustness of the obsolescence effect when adoption costs depend on the technical progress rate, and when they depend on the adopted technology. Section 8 concludes.

2 The model

We consider an economy with vintage capital and a Leontief technology. Technological progress is exogenous and we assume that is continuously embodied in the new capital goods. Consistently, we assume that our economy does not innovate, it simply adopts the most advanced technologies, however, it must exert an effort to adopt new technologies; in fact, we assume that adoption is costly and that it requires an extra amount of labor during a fixed interval of time, say D. The modelling of adoption through additional labor requirements is standard. In some recent contributions (as in Greenwood and Yorokoglu, 1997), adoption requires skilled labor. In others, as in Easterly et al. (1994), adoption costs are simply taken proportional to the labor force. In our setting, although we consider that adoption and production are two distinct activities, we do not introduce any skill differential in the model: labor is homogeneous and hence the labor market is not segmented (as it will appear very clearly later, our results will not be altered if a skill differential were to be introduced). More precisely, production and adoption are linked as follows: to form a production unit, a firm should combine one unit of capital, one unit of labor devoted to production (which is needed along the lifetime of the production unit) and α units of labor for D periods of time devoted to adoption. The embodied technological progress is assumed to be labor augmenting and increasing at a rate γ. Our specification can be interpreted as follows. There are two distinct phases in the lifetime of a production unit: for the first D periods an extra labor effort is needed to operate the capital goods which incorporate the latest technological advances. Once this phase is finished, the firm is assumed to have enough expertise to produce the same quantity of output with a lower amount of labor. It is worth pointing out that because of the underlying Leontieff structure, our setting is equivalent to assuming a decrease in labor productivity for the first D periods, which the firm compensates by hiring an extra amount of labor which is a standard specification in the recent literature of the field.

If we denote by $T(t)$ the age of the oldest operating machines at time t, aggregate employment $L(t)$ and output $Y(t)$ are given by:

$$Y(t) = \int_{t-T(t)}^{t} H(s) e^{\gamma s} ds$$

$$L(t) = \int_{t-T(t)}^{t} H(s) ds + \int_{t-D}^{t} \alpha H(s) ds$$

where $H(t)$ denotes the production units creation rate due to the Leontieff technology. As it will be clear later, the lifetime of production units, $T(t)$, is determined endogenously in the model in contrast to the adoption period D which is taken as constant. The assumptions underlying our specifications are obviously consistent with $T(t) > D$ for every t: the adoption phase cannot exceed the lifetime of the production units. Instead, in the spirit of the model, the lifetime of the production units should be significantly greater than the adoption time. We will return to this issue later.

We assume that the investment cost is linear and proportional to the leading productivity: $i(t) = H(t) e^{\gamma t}$. To close the model, we specify the consumer side:

the economy comprises a continuum of agents, indexed from 0 to 1. All individuals share the same linear preferences over lifetime consumption:

$$\int_0^\infty C(t)e^{-rt}dt$$

where $r > 0$ is the subjective rate of time preference, and $C(t)$ is the individual's consumption at time t; there is no desutility of labor, hence the supply is exogenous and equal to one.

2.1 The Central Planner Problem

The central planner solves the following problem:

$$\max \int_0^\infty C(t)e^{-rt}dt$$

subject to

$$\begin{aligned} C(t) &= Y(t) - i(t) \\ i(t) &= H(t)e^{\gamma t} \\ Y(t) &= \int_{t-T(t)}^{t} H(s)e^{\gamma s}ds \end{aligned} \tag{1}$$

$$\int_{t-T(t)}^{t} H(s)ds + \int_{t-D}^{t} \alpha H(s)ds = 1 \tag{2}$$

given $H_0(t)$ for all $t < 0$.

In order to solve this control problem we maximize the associated Lagrangian function; then, after changing the order the integration (following Malcomson (1975)), and some algebra, we get:

$$\begin{aligned} L(t) = {} & \int_0^\infty [Y(t) - H(t)e^{\gamma t} - \phi(t)Y(t) + \omega(t)]e^{-rt}dt + \\ & \int_0^\infty H(t)\left[\int_t^{t+J(t)}[e^{\gamma t}\phi(s) - \omega(s)]e^{-r(s-t)}ds - \alpha\int_t^{t+D}\omega(s)e^{-r(s-t)}ds\right]e^{-rt}dt + \\ & \int_{-T(0)}^{0} H(t)\left[\int_0^{t+J(t)}[e^{\gamma t}\phi(s) - \omega(s)]e^{-r(s-t)}ds - \alpha\int_0^{t+D}\omega(s)e^{-r(s-t)}ds\right]e^{-rt}dt \end{aligned}$$

where

$$J(t) = T(t + J(t))$$

The interior solution of this optimization problem is characterized by the following first order conditions:

$$
\begin{aligned}
\phi(t) &= 1 \quad \forall t \\
e^{\gamma t} &= \omega(t+J(t)) && (3)\\
e^{\gamma t} &= \int_t^{t+J(t)} \left[\phi(s)e^{\gamma t} - \omega(s)\right] e^{-r(s-t)} ds - \alpha \int_t^{t+D} \omega(s) e^{-r(s-t)} ds && (4)\\
J(t) &= T(t+J(t)) && (5)
\end{aligned}
$$

Where $\phi(t)$ and $\omega(t)$ are the multipliers associated with the constraints (1) and (2) respectively. Equation (3) is an exit condition which requires that the marginal value of labor $\omega(t)$ is equal to its marginal productivity. Equation (4) is an entry condition which equalizes the marginal cost of investment and the expected marginal revenue over its planned lifetime, $J(t)$. Equation (5) is just a definition: the expected lifetime of the new capital goods $J(t)$ is equal to the age of the oldest capital goods at time $t+J(t)$.

We are now able to define an equilibrium for our economy.

Definition 1 *Given the adoption time $D \geq 0$, and given the initial conditions $H_0(t)$, $\forall t<0$, an equilibrium for our economy is a path for $T(t)$, $J(t)$, $H(t)$ and $Y(t)$, $\forall t \geq 0$, such that $T(t)$ and $J(t)$ are strictly greater than $D \geq 0$, and satisfies the system of equations:*

$$1 = \int_t^{t+J(t)} \left[1 - e^{-\gamma(t-s+T(s))}\right] e^{-r(s-t)} ds - \alpha \int_t^{t+D} e^{-\gamma(t-s+T(s))} e^{-r(s-t)} ds \qquad (6)$$

$$J(t) = T(t+J(t)) \qquad (7)$$

$$1 = \int_{t-T(t)}^{t} H(s) ds + \int_{t-D}^{t} \alpha H(s) ds$$

$$Y(t) = \int_{t-T(t)}^{t} H(s) e^{\gamma s} ds$$

3 Optimal Scrapping and Adoption Time

Our equilibrium conditions show a clear recursive forward-looking sub-block, namely the sub-block formed by equations (6)-(7). This sub-block allows to solve for the timing variables $T(t)$ and $J(t)$ independently of the other endogenous variables. By differentiating (6), using (7) and rearranging terms, we find the following functional relation

$$
\begin{aligned}
T(t) &= F(T(t+D); J(t)) \\
&= -\frac{1}{\gamma} \ln \left[\frac{1}{1+\alpha} \left(1 - (r-\gamma) - \frac{\gamma}{r}(1 - e^{-rJ(t)}) + \alpha e^{-\gamma T(t+D)} e^{-(r-\gamma)D} \right) \right]
\end{aligned}
$$

In order for function $F(.;.)$ to map from $R_+ \times R_+$ into R_{++}, we need the following assumption:

Assumption 1 The parameters of the model must satisfy the following conditions: $0 < \gamma < r < 1$.

This assumption on the parameters is a standard condition for the existence of solutions in exogenous growth models; it is not difficult to prove that under assumption 1, (i) function $F(.;.)$ is increasing with respect to each of its arguments, and (ii) function $G(x) = F(x;x)$ has a unique strictly positive fixed-point. The following existence-uniqueness result generalizes Boucekkine, Germain and Licandro's Proposition 2 (1997).

Proposition 2 *Under assumption 1, for any value of the adoption time $D \geq 0$, the unique equilibrium paths for $T(t)$ and $J(t)$, $t \geq 0$, are constant and equal to the fixed-point T^* of the function $G(x) = F(x;x)$.*

Proof. The proof consists in constructing a sequence of upper bounds and lower bounds for $T(t)$, $\forall t$, and to show that these two sequences converge to the fixed-point of function $G(x)$. Recall that

$$
\begin{aligned}
T(t) &= -\frac{1}{\gamma} \ln \left[A + B e^{-rJ(t)} + C e^{-\gamma T(t+D)} \right] \\
A &= \frac{r-\gamma}{1+\alpha} \left(\frac{1}{r} - 1 \right) > 0 \\
B &= \frac{\gamma}{r(1+\alpha)} > 0 \\
C &= \frac{\alpha}{1+\alpha} e^{-(r-\gamma)D} > 0
\end{aligned}
$$

An obvious lower bound for $T(t)$ is $F(0;0) = -\frac{1}{\gamma} \ln[A+B+C]$, and a simple upper bound is $F(\infty;\infty) = -\frac{1}{\gamma} \ln[A]$. We get

$$
-\frac{1}{\gamma} \ln[A+B+C] \leq T(t) \leq -\frac{1}{\gamma} \ln[A]
$$

for all $t \leq 0$. Since the previous inequalities hold for all t, they hold at $t+D$ and at $t+J(t)$. As $T(t+J(t)) = J(t)$, we can find another lower bound and another upper bound for $T(t)$ using the fact that function $F(.,.)$ is increasing in each of its arguments:

$$-\frac{1}{\gamma}\ln\left[A+Be^{\frac{r}{\gamma}\ln[A+B+C]}+Ce^{\ln[A+B+C]}\right]$$
$$\leq T(t)\leq -\frac{1}{\gamma}\ln\left[A+Be^{\frac{r}{\gamma}\ln A}+Ce^{\ln[A]}\right]$$

We can keep on generating successive lower bounds (a_n) and upper bounds (b_n) for $T(t)$ in this way. For lower bounds we get the sequence $a_0 = -\frac{1}{\gamma}\ln[A+B+C]$ and $a_n = -\frac{1}{\gamma}\ln[A+Be^{ra_{n-1}}+Ce^{\gamma a_{n-1}}] = G(a_{n-1})$, for all $n \geq 1$. For upper bounds we get the sequence $b_0 = -\frac{1}{\gamma}\ln[A]$ and $b_n = -\frac{1}{\gamma}\ln\left[A+Be^{rb_{n-1}}+Ce^{\gamma b_{n-1}}\right] = G(b_{n-1})$. The sequence a_n (b_n) is trivially increasing (decreasing) and bounded. Thus, both sequences converge, they converge to the fixed point of function $G(.)$ by construction. Proposition 1 follows immediately. ■

Note that the constancy of the optimal (interior) lifetime of production units is obtained even in the zero adoption cost case, namely if $D = 0$. This property is useful for comparison purposes. A first interesting economic insight can be gained from the study of optimal lifetime when the adoption time varies.

Proposition 3 *Under assumption 1, the optimal lifetime of production units T^* is an increasing but concave function of the adoption time D.*

Proof: See the proof in the Appendix. ■

The proposition shows two interesting economic properties of the model. First, adoption costs increase the lifetime of production units and delay replacement of the oldest machines. Secondly, the growth rate of the production's units lifetime is decreasing with respect to the adoption time. This is rather an expected property having in mind equilibrium equation (6). An increase in the adoption time will increase the associated labor cost, which tends to increase the lifetime variable $J(t)$ as it requires time to recover the additional labor costs. However, an increase in the lifetime variable $J(t)$ decreases the "shadow" wages by equation (3), so it will reduce ex-post the labor costs. As a result, the optimal lifetime tends to growth less than the adoption time.

Indeed, the later issue as a whole is not that important in our simple framework since the adoption time is exogenous and can be fixed to a convenient value consistently with the view underlying our specification of adoption. Recall that, by definition of equilibrium, the optimal lifetime of production units should be lower than the adoption time. The concavity result above implies that the latter desired property may not be obtained for large values of D. We can prevent such an undesirable configuration by restricting the values of adoption time. We can establish the following existence result.

Proposition 4 *Under assumption 1, there exists $D_0 > 0$ such that $T^* > D$ if and only if $0 \leq D \leq D_0$*

The previous proposition restricts the value of the adoption time under which $T(t) = T^*$ is an equilibrium for all $t \geq 0$. We think of adoption time as a short transition period compared to the whole lifetime of production units. In this context, proposition 4 works extremely well.[2]

4 Steady State and Comparative Statics

This section characterizes the steady-state and analyzes the effects of changes in the rate of technical progress, γ, and the adoption costs $(\alpha; D)$ on the long run values of the variables of the model. The steady state of this economy is a situation in which the rate of job creation, detrended production and the optimal life of the machines are constants, i.e., $H(t) = H$, $Y(t) = Ye^{\gamma t}$, $T(t) = T(t + D) = J(t) = T^*$. It is characterized by the following equations:

$$1 = \frac{1 - e^{-rT^*}}{r} - \frac{e^{-\gamma T^*} - e^{-rT^*}}{r - \gamma} - \frac{\alpha e^{-\gamma T^*}(1 - e^{-(r-\gamma)D})}{r - \gamma} \tag{8}$$

$$1 = HT^* + \alpha HD \tag{9}$$

$$Y = \frac{H(1 - e^{-\gamma T^*})}{\gamma} \tag{10}$$

Using the equations above, we can establish the following comparative statics results, which are obtained by differentiating the equations above (see the appendix).

(i) $\frac{\partial T^*}{\partial D} > 0; \quad \frac{\partial T^*}{\partial \alpha} > 0; \quad \frac{\partial T^*}{\partial \gamma} < 0$

An increase in the adoption period, or an increase in the additional labor required to implement the new technology raises the associated labor costs and augments the required time to recover the investment; consequently adoption costs increase the optimal lifetime of the machines.

The inverse relation between T^* and γ is standard in vintage capital models; it is the typical obsolescence effect, reflecting that an increment in the rate of technological progress makes replacement more profitable. As adoption costs are constant, the obsolescence effect is not affected by them. In subsequent sections, we will show that this result no longer holds when we relax the constant adoption costs assumption.

[2] For example, if $r = 0.05$, $\gamma = 0.03$, $\alpha = 0.2$ and $d = 3$, T^* equals to 11.25 years, while d_0 is 16.7 years.

(ii) $\frac{\partial H}{\partial D} < 0;\quad \frac{\partial H}{\partial \alpha} > 0,\quad \frac{\partial H}{\partial \gamma} > 0$

A rise in the adoption costs should decrease the job creation rate; in fact, there are two negative effects. First, there is a direct effect: adoption costs increase labor costs and discourage investment. Second, there is an indirect effect which reinforces the direct one: adoption costs augment the optimal lifetime T^*, and so job creations should diminish to maintain the labor market equilibrium.

On the other hand, an increase in the rate of technical progress raises the rate of job creation. Faster technical progress decreases the optimal age of the machines and consequently, in equilibrium, job creation must increase.

(iii) $\frac{\partial Y^*}{\partial D} > 0;\quad \frac{\partial T^*}{\partial \alpha} > 0;\quad \frac{\partial Y}{\partial \gamma} < 0$

An increment in the adoption costs has two opposing effects on detrended output. First, it increases the optimal lifetime and tends to raise detrended output. Second, there is a negative effect, working through a reduction of the job creation rate, pushing toward an increase in detrended output. As we show in the Appendix, the total effect is unambiguously negative.

An increase in γ has three competing effects on detrended output:

$$\frac{\partial Y}{\partial \gamma} = \underset{(+)}{\frac{1 - e^{-\gamma T}}{\gamma}\frac{\partial H}{\partial \gamma}} + \underset{(-)}{H e^{-\gamma T}\frac{\partial T}{\partial \gamma}} + \underset{(-)}{\frac{H(\gamma T e^{-\gamma T} - 1 + e^{-\gamma T})}{\gamma^2}}$$

First, there is a positive effect which works in the direction of increasing the equilibrium job creation rate, and hence augmenting detrended output. Second, a rise in the rate of technical progress decreases the optimal lifetime and has a negative effect on detrended output. Finally, taken H and T as given, an increase in γ reduces the long run output. Which of the conflicting effects will dominate will depend upon the size of them. We show in the appendix that the total effect is negative and does not depend on the values of the parameters of the model.

5 Dynamics and Asymptotic Properties of Job Creation and Production

In this section, we will study the short run dynamics and the asymptotic stability of job creation and production. As there is no unemployment in the model, the dynamics of job creation rate $H(t)(1+\alpha)$ are identical to those of job destruction rate $H(t - T^*) + \alpha H(t - D)$.

Differentiating equation (9), the labor market equilibrium equation:

$$H(t) = aH(t - D) + (1 - a)H(t - T^*) \tag{11}$$

where $a = \frac{\alpha}{1+\alpha}$. To derive the dynamics of production, we differentiate equation (1) and, taking into account equation (10), obtain:

$$Y(t) = aY(t - D)e^{\gamma D} + (1 - a)Y(t - T^*)e^{\gamma T^*} \tag{12}$$

In the case where adoption costs are zero, the RVCM studied by Boucekkine et al (1996), the dynamics of job creation and production are purely periodic in the short and in the long run.

$$\begin{aligned} H(t) &= H(t - T^*) \\ Y(t) &= Y(t - T^*)e^{\gamma T^*} \end{aligned}$$

The main implication of adoption costs is the appearance of a second delay in the dynamics of job creation. This second delay is also obtained in the presence of a skill differential between the production and adoption activities under the assumption that the two activities have not identical timing, which is why we argue above that the introduction of skill differentials is not necessary for the dynamics of our model: the introduction of adoption costs will distort the equilibrium dynamics as long as they involve a different timing with respect to the main creation and destruction decisions. As the dynamics of $Y(t)$ are identical to those of $H(t)$, we will focus on the dynamics of $H(t)$.

Proposition 5 *All the nonzero roots of (11) are stable*

Proof. The characteristic function associated to equation (11) is

$$g(\lambda) = 1 - ae^{-\lambda D} - (1 - a)e^{-\lambda T^*} = 0 \tag{13}$$

Let $\lambda = x + iy$, then $g(\lambda) = 0$ implies that the real and imaginary part of (13) must be zero.

$$\begin{aligned} 1 - ae^{-xD}\cos(xD) - (1 - a)e^{-xT^*}\cos(xT^*) &= 0 \quad (14) \\ ae^{-xD}\sin(yD) + (1 - a)e^{-xT^*}\sin(yT^*) &= 0 \quad (15) \end{aligned}$$

First, we prove that $g(\lambda)$ has a non-positive real part; we then check that if the real part is zero, the imaginary part is also zero. It is easy to see that $x > 0$ is impossible since it implies $ae^{-xD}\cos(xD) + (1 - a)e^{-xT^*}\cos(xT^*) < 1$, ($e^{-z}$ and $\cos t \le 1$, for any t, and for any $z > 0$). On the other hand, if $x = 0$, equations (14) and (15) can be written as

$$\begin{aligned} 1 - a\cos(xD) - (1 - a)\cos(xT^*) &= 0 \\ a\sin(yD) + (1 - a)\sin(yT^*) &= 0 \end{aligned}$$

and it is trivial to check that the only value of y that satisfies the previous equations is $y = 0$. So, the unique real root is the trivial root $\lambda = 0$. ■

The previous proposition gives us the following result for the short run and asymptotic behavior:

Corollary 6 (i) Unless the initial condition function $H_0(t)$ is constant and equal to H, the dynamics of job creation have a cyclical, but "asymmetrical" behavior in the short run;

(ii) For all $H_0(t)$,

$$\lim_{t \to \infty} H(t) = H = \frac{1}{T^* + \alpha D}$$

The previous corollary is a direct result of equation (11) and proposition 5. Given that the characteristic function associated to equation (11) has only $\lambda = 0$ as a real root, the dynamics of job creation are asymptotically stable and $H(t)$ converges to its steady value.

In contrast with the zero adoption costs model we have obtained two main results. First, when the adoption costs are positive, fluctuations vanish in the long run. Second, convergence to the steady state is cyclical, but shows some irregular patterns; this asymmetric behavior is a consequence of the existence of a second delay, that is, the adoption time, and depends on the size of the adoption costs.

We analyze equation (11) to see the role α and D play in the short run dynamics of job creation. In fact, $H(t)$ is a linear combination of $H(t - D)$ and $H(t - T^*)$. The parameter $a = \frac{\alpha}{1+\alpha}$ is an increasing function of α. Given D, the adoption labor requirement measures the effect of the adoption delay in $H(t)$; if α is large (small), the dynamics of job creation is dominated by the job creation D (T^*) periods before. On the other hand, given α, the adoption time gives the frequency of the irregular patterns. The irregular behavior of $H(t)$ depends crucially on the initial conditions. As we show, if we establish an increasing (decreasing) initial conditions, the gap between $H(t - D)$ and $H(t - T^*)$, and the irregular pattern is bigger than in the case in which the initial conditions are cyclical. Some examples of the previous result are given in the appendix.

In the simple economy which we have just analyzed, technical progress accelerates the technological substitution process, given the adoption costs. To deal with the obsolescence effect in a wide framework, we extend the model in two ways. Firstly, we assume that adoption costs depend on the technical growth rate. Secondly, we endogenize the adoption cost to allow to adoption costs to depend on the adopted technology. In the latter approach, we show that we can deal with the use of dominated technologies and analyze the role of technical progress in the technological gap.

6 Adoption Costs and Technical Progress

In the previous section, we assumed exogenous and constant adoption costs which is equivalent to incorporating a fixed cost. This is simple and unrealistic modeling. In fact, the process of implementing new technologies is uncertain in nature and the best way to model it is from a Bayesian perspective (Jovanovic and Nyarko, 1995). However trying to do this in a general equilibrium framework is a daunting task, and general equilibrium models deal with this problem assuming a mechanistic process. Greenwood and Yorokoglu (1996) stress the difficulty of modelling the adoption process in an endogenous manner, and assume that as the rate of technological progress increases, the more costly it becomes to adopt the new technology since enterprises will be less familiar with it.

In this section, we consider the same approach as Greenwood and Yorokoglu (1996) and assume that adoption costs depend positively on the rate of technical progress. With this assumption, faster growth rates increase the adoption cost. This is a suitable framework to study whether countries with costly adoption processes delay the technological substitution and use machines that other countries (with lower adoption costs) have already replaced. Parente and Prescott (1994) show that the disparity in technology adoption barriers (which can be captured by our adoption costs) account for the huge observed income disparity across countries. Our objective is to analyze whether the disparity in adoption costs can explain the use of different technologies across countries. We have pointed out that in vintage capital models with zero or constant adoption costs there is an obsolescence effect which makes the replacement of the oldest machines profitable. In this section we study the robustness of this result in a context where the rate of technological progress affects the adoption costs. As we see, the technological progress can hold up machine replacement if the adoption costs are sufficiently large.

We consider the following adoption costs function, $C(\alpha(\gamma); D)$; modelling in this way implies that the adoption period is constant. We assume that the labor requirement to implement the new technology depends on the rate of technical progress.

Assumption 2 $\alpha'(\gamma) > 0$, and $\alpha''(\gamma) > 0$.

Assumption 2 states that the adoption cost function is increasing and convex in γ. It implies that faster technical progress increases the cost of the adoption process. The convexity assumption is required to reflect that faster technical growth rates lead to larger differences between technologies of different vintages, and so, implementation of the latest technologies is costly.

Proposition 7 1) If the technical progress rate γ, the additional labor requirement

α, or the duration of the adoption phase D, are sufficiently small, then $\frac{\partial T^*}{\partial \gamma}$ becomes strictly negative.

2) If the adoption process is sufficiently costly, $\exists\, \gamma'$ so that $\frac{\partial T^*}{\partial \gamma} > 0$ $\forall \gamma > \gamma'$. Besides, γ' is a decreasing function in D.

Proof: We differentiate equation (8) taking into account that α depends on the rate of technological progress:

$$\frac{\partial T^*}{\partial \gamma} = \frac{1 - e^{-(r-\gamma)T^*} - T^*(r-\gamma) + \alpha\left[1 - e^{-(r-\gamma)D}\left(1 + (r-\gamma)(D - T^*)\right) - T^*(r-\gamma)\right] + (r-\gamma)\alpha'(\gamma)\left(1 - e^{-(r-\gamma)D}\right)}{(r-\gamma)\gamma\left[1 - e^{-(r-\gamma)T^*} + \alpha\left(1 - e^{-(r-\gamma)D}\right)\right]}$$

We can express the previous expression in terms of $C(\alpha(\gamma); D)$:

$$\frac{\partial T^*}{\partial \gamma} = \underset{(-)}{\left.\frac{\partial T^*}{\partial \gamma}\right|_c} + \underset{(+)}{\frac{\partial T^*}{\partial \alpha(\gamma)} * \frac{\partial \alpha(\gamma)}{\partial \gamma}} \tag{16}$$

In contrast with the typical obsolescence effect which we obtained with constant adoption costs, technological progress also affects the optimal lifetime through the adoption costs in this framework. As we can see, equation (13) clearly shows two opposing effects.

First, there is a direct destruction creative effect which decreases the age of capital. Given the adoption costs, an increase in the technical growth rate implies faster obsolescence and a decrease the age of capital. The direct destruction creative effect decreases with the technological progress rate, due to the fact that $T^*(\gamma)$ is a decreasing and convex function.

Second, there is an adoption effect, which increases the life of capital: an increase in the growth rate raises the additional labor requirement and increases the optimal lifetime of machines as it requires time to recoup the larger adoption costs.

$$\text{Adoption effect} = \frac{\partial T^*}{\partial \alpha}\alpha'(\gamma) = \frac{(r-\gamma)\alpha'(\gamma)\left(1 - e^{-(r-\gamma)D}\right)}{(r-\gamma)\gamma\left[1 - e^{-(r-\gamma)T^*} + \alpha\left(1 - e^{-(r-\gamma)D}\right)\right]}$$

The previous effect depends mainly on adoption costs; the duration of the adoption period and the effect of γ on α have a positive effect on the size of the adoption effect.

The total effect is ambiguous and it depends on γ, $\alpha(\gamma)$ and D ■

Proposition 7 reflects the fact that technological progress can hold up the machine replacement when the adoption costs depend on the technical progress rate, and when they are sufficiently large. If we think of adoption costs as different institutional barriers (Parente and Prescott, 1994) across countries, proposition 7 produces an interesting result: in countries with small adoption costs the obsolescence effect compensates the adoption effect and faster growth rates accelerate technological sub-

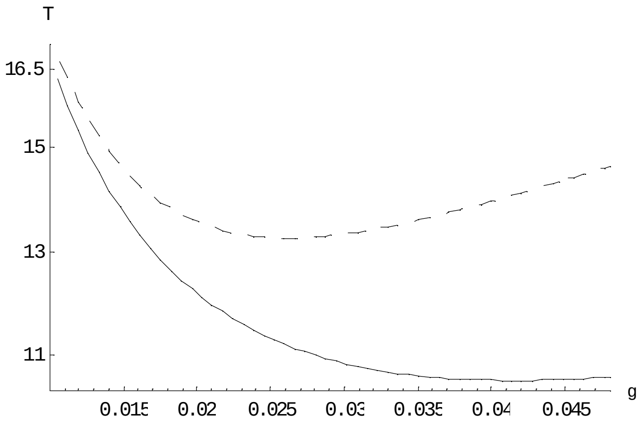

Figure 1. — C_1, - - -C_2

stitution. However, in countries with larger adoption costs the predominant effect is the adoption one and technical progress discourages machine replacement which leads to technological sclerosis.

Example.

We deal with a numerical example to compare two economies with different adoption costs functions: $(C_i(\alpha_i(\gamma); D_i)$, $i = 1; 2$ (figure 1). According to Greenwood and Yorokoglu (1997), adoption costs may amount to about 10% of GDP in developed countries. We choose adoption functions, so that, in terms of Greenwood and Yorokoglu (1997), they reflect a developed and a developing economies. We set the following adoption costs functions $\alpha_i(\gamma) = a_i \gamma^b$ and the parameters of the model are supposed to be $r = 0.05$, $D_1 = 3$ years, $D_2 = 6$ years, $a_1 = 1000$, $a_2 = 1500$, $b = 2.5$. In the first economy, with $\gamma = 3\%$, the amount of labor devoted to implementing the technology is $\alpha_1 = 15\%$, and the ratio of adoption costs to production does not exceed 10%; we define this country as a developed economy. On the contrary, with $C_2(\alpha_2(\gamma); D_2))$, $\alpha_2 = 23\%$, and the ratio of adoption costs to production is about 13%; we define this country as a developing economy.

In the developed economy an increase in the rate of technical progress diminishes the optimal lifetime; the obsolescence effect is bigger than the adoption effect regardless of the value of γ. In contrast, in the developing economy the obsolescence effect predominates for low values of γ, but, as the technical progress rate increases, the predominate effect is the adoption one: increasing γ delays the technological substitution.

Note that the size of the first derivative of the adoption costs function is crucial for proposition 7 (the adoption effect tends to zero when $\alpha'(\gamma)$ tends to zero). Further numerical examples show that the bigger $\alpha'(\gamma)$, the smaller is the adoption period and the technical growth rate required to break down the obsolescence effect.

7 Endogenous Adoption Costs

In this section we study the machine replacement process when adoption costs depend on the adopted technology and there is a trade-off between adopting the latest technology and the adoption costs incurred to implement it. In the previous sections it is always profitable to adopt the latest technology because adoption costs are independent of the quality of the technology. It is well known that developing economies invest in dominated technologies, because their implementation are cheaper, and the quality investment decision therefore creates a productivity gap between the leader and the adopted technology. Modeling adoption costs in an endogenous way allows us to extend the model to deal with the use of dominated technologies. We will show that when economies can control adoption costs by choosing the adopted technology as a decision variable, we recoup the obsolescence effect, and greater technical growth rates reduce the optimal lifetime of the capital goods. On the contrary, we will obtain that technological progress has a negative effect on the technological gap.

Adoption costs should depend on the difference between the adopted technology and the scrapped one; however, this assumption makes the solution of the model considerably more difficult. We deal with this problem by assuming that the adoption costs depends on the difference between the leader and the adopted technology.. This is a reasonable assumption as it implies that what is really costly is adopting the newest technologies; the underlying idea is that technological use along time becomes common knowledge, and using older technologies is cheaper than using newer ones, independently of the technological starting point.

We denote the technological delay in t by $K(t)$; that is, in period t, the economy adopts the capital goods created in $t-K(t)$; with an associated productivity $e^{\gamma(t-K(t))}$. We assume that the adoption period, D, is constant, and the labor requirement devoted to adoption depends on the technological delay, $\alpha(K(t))$. We assume that the investment cost is proportional to the acquired technology.

The central planner now has to solve the following problem

$$\max \int_0^\infty [Y(t) - i(t)] e^{-rt} dt \tag{17}$$

subject to

$$Y(t) = \int_{t-T(t)}^{t} H(s) e^{\gamma(s-K(s))} ds \tag{18}$$

$$1 = \int_{t-T(t)}^{t} H(s) ds + \int_{t-D}^{t} \alpha(K(s)) H(s) ds \tag{19}$$

$$i(t) = H(t) e^{\gamma(t-K(t))}$$

given $H_0(t)$ for all $t < 0$. This problem is equivalent to the central planner's problem solved in section 2, yet in this case we have an additional endogenous variable: $K(t)$. The associated Lagrangian function is:

$$L(t) = \int_0^{\infty} [Y(t) - H(t)e^{\gamma(t-K(t))} - \lambda(t)Y(t) + \omega(t)]e^{-rt}dt + \int_0^{\infty} H(t)\left[\int_t^{t+J(t)}\left[e^{\gamma(t-K(t))}\lambda(s) - \omega(s)\right]e^{-r(s-t)}ds - \alpha(K(t))\int_t^{t+D}\omega(s)e^{-r(s-t)}ds\right]e^{-rt}dt + \int_{-T(0)}^{0} H(t)\left[\int_0^{t+J(t)}\left[e^{\gamma(t-K(t))}\lambda(s) - \omega(s)\right]e^{-r(s-t)}ds - \alpha(K(t))\int_0^{t+D}\omega(s)e^{-r(s-t)}ds\right]e^{-rt}dt$$

where $\lambda(t)$ and $\omega(t)$ are the Lagrangian multipliers associated with constraints (17) and (18) respectively. After some algebra and rearranging terms, the interior solution is characterized by the following first order conditions:

$$1 = \int_t^{t+J(t)}\left[1 - e^{-\gamma(t-s-K(t)+K(s)+T(s))}\right]e^{-r(s-t)}ds - \alpha(K(t))\int_t^{t+D} e^{-\gamma(t-s-K(t)+K(s)+T(s))}e^{-r(s-t)}ds \tag{20}$$

$$1 = \int_t^{t+J(t)} e^{-r(s-t)}ds + \frac{\alpha'(K(t))}{\gamma}\int_t^{t+D} e^{-\gamma(t-s-K(t)+K(s)+T(s))}e^{-r(s-t)}ds \tag{21}$$

$$J(t) = T(t + J(t)) \tag{22}$$

Equation (20) is an entry condition, and equation (21) is the optimal condition for K; it can be written as

$$\gamma\int_t^{t+J(t)} e^{-r(s-t)}ds = \gamma - \alpha'(K(t))\int_t^{t+D} e^{-\gamma(t-s-K(t)+K(s)+T(s))}e^{-r(s-t)}ds$$

and requires that the marginal revenue of decreasing the adoption lag equalizes the marginal cost of investment and the additional adoption costs (we later assume that $\alpha(K(t))$ is decreasing function of K). We restrict our analysis to the stationary equilibrium for our economy.

Definition 8 Given the adoption time $D \geq 0$, and given the initial conditions $H(t); \forall t < 0$, a stationary equilibrium for our economy is a situation in which the rate of job creation, detrended output, the optimal life of the machines and the technological delay are constants, i.e., $H(t) = H$, $Y(t) = Ye^{\gamma t}$, $T(t) = J(t) = T$, $K(t) = K$.

It is characterized by the following equations:

$$
\begin{aligned}
1 &= \frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} - \frac{\alpha(K)e^{-\gamma T}(1 - e^{-(r-\gamma)D})}{r - \gamma} && (23)\\
1 &= \frac{1 - e^{-rT}}{r} + \frac{\alpha'(K)e^{-\gamma T}(1 - e^{-(r-\gamma)D})}{(r - \gamma)\gamma} && (24)\\
Y &= \frac{He^{-\gamma K}(1 - e^{-\gamma T})}{\gamma} && (25)\\
1 &= HT + \alpha(K)HD && (26)
\end{aligned}
$$

The problem of checking the existence of a stationary equilibrium is reduced to verifying that $(T; K)$ exists, which solves equations (23) and (24). To study the existence of a stationary equilibrium we need to make some assumptions about adoption costs.

Assumption 3

(i) $K \in [0; K_{max}]$; and K_{max} is such that $\alpha(K_{max}) = 0$

(ii) $\alpha'(K) < 0$, $\alpha''(K) < 0$. Furthermore, $\alpha(0) \to \alpha_0$ and $\alpha'(0) \to 0$;

The previous assumption implies that the labor requirement devoted to adoption cannot be negative, and restricts the behavior of $\alpha(K)$ in a reasonable way. The adoption costs are decreasing in the technological gap, and the effect of the technological gap as reducing the adoption costs is increasing in the gap size. In other words, the costs of adopting technologies diminishes at an increasing rate with the technological gap. We denote the additional labor requirement when the firm acquires the leader technology by α_0.

Proposition 9 *Under assumption 3, a stationary equilibrium exists for our economy. The equilibrium is therefore unique.*

Proof: We first prove that equation (23) defines T as a differentiable and decreasing function of K, and equation (24) defines T as a differentiable and increasing function of K,

$$
\begin{aligned}
F(T; K) &= \frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} - \frac{\alpha(K)e^{-\gamma T}(1 - e^{-(r-\gamma)D})}{r - \gamma} - 1 = 0 && (27)\\
G(T; K) &= \frac{1 - e^{-rT}}{r} + \frac{\alpha'(K)e^{-\gamma T}(1 - e^{-(r-\gamma)D})}{\gamma(r - \gamma)} - 1 = 0 && (28)
\end{aligned}
$$

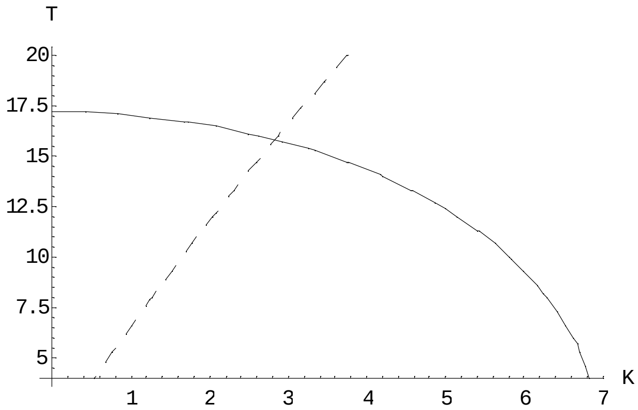

Figure 2. – $F(T;K)$, - - - $G(T;K)$

If we denote the functional relation between T and K given by $F(T;K)$ ($G(T;K)$) by $f(\cdot)$ ($g(\cdot)$)

$$f'(T) = \frac{\alpha'(K)(1-e^{-(r-\gamma)D(K)})}{\gamma[1-e^{-(r-\gamma)T}+(1-e^{-(r-\gamma)D})\alpha(K)]} < 0$$

$$g'(T) = -\frac{\alpha''(K)(1-e^{-(r-\gamma)D})}{\gamma(r-\gamma)e^{-(r-\gamma)T}-\alpha'(K)(1-e^{-(r-\gamma)D})} > 0$$

When the technological delay tends to zero we can easily show that $T^f > T^g$; in fact,

$$F(T;0) = \frac{1-e^{-rT}}{r} - \frac{e^{-\gamma T}-e^{-rT}}{r-\gamma} - \frac{\alpha_0 e^{-\gamma T}(1-e^{-(r-\gamma)D})}{r-\gamma} - 1 = 0$$

$$G(T;0) = \frac{1-e^{-rT}}{r} - 1 = 0$$

Summing up, $T = f(K)$ ($T = g(K)$) defines a decreasing (increasing) function, and f is above g when K tends to zero; so, they only intercept once, and the steady state is hence unique. ■

A stationary equilibrium is any set $(T;K;H;Y)$ that solves equations (23)-(26). Because the relation defined by $F(T;K)$ is downward sloping as illustrated in figure 2, while $G(T;K)$ is upward sloping, there is a single equilibrium solution to the two equations. Then, there is a single solution to equations (25) and (26), and the steady state equilibrium is unique.

Figure 2 provides insight into how the various parameters of the model affect the steady-state optimal lifetime of the machines and technological gap. Note that the technological gap, defined as the ratio between the leader technology and the adopted one, is equal to $e^{\gamma K}$.

An increase in the labor requirement devoted to adoption when the technological gap is zero, α_0, should increase the lifetime of machines and the technological gap. An increase in α_0 shifts the F curve up but has no effect on G. Consequently the equilibrium value of the optimal lifetime, the technological delay and the technological gap increase with α_0. The increment in the technological gap to reduce the labor requirement to adoption reduces the productivity of the adopted technology, increasing the obsolescence effect and inducing a shorter lifetime of the machines. On the other hand, α_0 can be understood as a fixed cost and thus augments the optimal age of the machines as it requires more time to recoup the additional adoption cost. The total effect is positive, and in the long run T rises with an increase in α_0.

An increase in D raises the optimal lifetime, the effect on the technological gap is however ambiguous. A higher D shifts the F curve up and the K curve to the left. Once again, T increases to recoup the additional costs; the effect on K is ambiguous as there are two opposing effects: a higher D tends to increase K in order to reduce adoption costs, but, a higher T must be compensated by a smaller K in order for it to be profitable to keep the machine for a longer period of time.

An increase in the rate of technical progress shifts the F curve down and the G curve to the left, and so implies a lower T, but an ambiguous effect on K. A higher γ makes the replacement more profitable and reduces T. It also has two opposite effects on K. Firstly, it increases the reservation wage and thus increases the adoption costs, inducing an increment in K; secondly, a rise in γ leads to an increase in the productivity of the machines and stimulates the adoption of better technologies to take advantage of the productivity improvement. A formal differentiation of the equilibrium condition yields a negative effect on T, and an ambiguous effect on K; extensive experiments with the parameters give a positive effect on the technological delay and so, on the technological gap.

To summarize, in this section we show that the model is suitable to analyze the use of dominated technologies. Even in this simple framework the analysis of the dynamics to the steady state is not straightforward: it requires solving equations (20)-(22), a system of differential-difference equations with state dependent leads and lags. But, restricting the analysis to the steady state we obtain that with endogenous adoption costs the effect of technical progress on the optimal lifetime is negative, we recover the obsolescence effect inherent in vintage capital growth model. However, technical progress has a negative consequence on the technological gap, and faster technical growth rates lead to the adoption of older technologies.

8 Conclusions

In this paper we incorporate adoption costs in the Ramsey growth vintage capital model with linear utility. The aim of the paper is to analyze the consequences of adoption costs in technological substitution and in the dynamics of the model. In contrast with the zero adoption costs model, we obtain that fluctuations vanish in the long run and that the convergence to the steady state is cyclical and shows some irregular patterns. On the other hand, the obsolescence effect is a well known feature of creative destruction models: faster growth rates make capital goods less profitable and induce faster machine replacement. We introduce adoption costs in three reasonable ways and find that the obsolescence effect can be held up when adoption costs depend on the technical growth rate. This is an interesting result stressing that developing countries with higher adoption costs delay technological substitution. Finally, when we let economies adopt dominated technologies, we recover the obsolescence effect but we verify that technical progress increases the technological gap between developing and developed countries; in other words, developing countries deal with higher adoption costs investing in older technologies which require a smaller adoption effort.

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10 Appendix

Proof of Proposition 3

Total differentiation of the fixed-point equation $T^* = F(T^*; T^*) = G(T^*)$, after rearranging terms in this way

$$1 = \frac{1 - e^{-rT^*}}{r} - \frac{e^{-\gamma T^*} - e^{-rT^*}}{r - \gamma} - \frac{\alpha e^{-\gamma T^*}(1 - e^{-(r-\gamma)D})}{r - \gamma}$$

yields:

$$\frac{\partial T^*}{\partial D} = \frac{\alpha(r-\gamma)e^{-(r-\gamma)D}}{\gamma[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})]} > 0$$

Under assumption 1, the optimal lifetime increases if the adoption costs rise through an increase in D. To establish concavity, much more tedious computations are needed. Let us define the function $Q(.;.)$ in the following way:

$$Q(D; T^*) = \frac{1 - e^{-rT^*}}{r} - \frac{e^{-\gamma T^*} - e^{-rT^*}}{r - \gamma} - \frac{\alpha e^{-\gamma T^*}(1 - e^{-(r-\gamma)D}}{r - \gamma} - 1 = 0$$

then if we denote by $f(.)$ the functional relation between T^* and D, $T^* = f(D)$, the necessary second order derivative should satisfy

$$f''(D) = -\frac{1}{(Q_2)^3}\left[Q_{11}Q_2^2 + Q_{22}Q_1^2 - 2Q_1Q_2Q_{12}\right]$$

with

$$Q_1 = -\alpha e^{-\gamma T^*} e^{-(r-\gamma)D} < 0$$

$$Q_2 = \frac{\gamma e^{-\gamma T^*}\left[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D}\right]}{r - \gamma} > 0$$

$$Q_{11} = \alpha(r - \gamma)e^{-\gamma T^*} e^{-(r-\gamma)D} > 0$$

$$Q_{12} = \alpha\gamma e^{-\gamma T^*} e^{-(r-\gamma)D} > 0$$

$$Q_{22} = \frac{-\gamma^2 e^{-\gamma T^*}\left[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D}\right]}{r - \gamma} +$$

$$\gamma e^{-\gamma T^*} e^{-(r-\gamma)D}$$

Now observe that

$$Q_{22}Q_1^2 - 2Q_1Q_2Q_{12} = \alpha^2\gamma e^{-3\gamma T^*}e^{-2(r-\gamma)D}\left[e^{-(r-\gamma)T^*} + \frac{\left[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})\right]}{r-\gamma}\right] > 0$$

As $Q_{22}Q_1^2 > 0$ and $Q_{22}Q_1^2 - 2Q_1Q_2Q_{12} > 0$, we do get $f'(D) < 0$

Section 4. Comparative Static Results

(i) Differentiating equation (8) with respect to the adoption costs gives:

$$\frac{\partial T^*}{\partial D} = \frac{\alpha(r-\gamma)e^{-(r-\gamma)D}}{\gamma[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})]} > 0$$

$$\frac{\partial T^*}{\partial \alpha} = \frac{1 - e^{-(r-\gamma)D}}{\gamma[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})]} > 0$$

Differentiating equation (8) with respect to γ, we obtain

$$\frac{\partial T^*}{\partial \gamma} = \frac{1 - e^{-(r-\gamma)T^*} - T^*(r-\gamma)}{(r-\gamma)\gamma[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})]} + \frac{\alpha\left[1 - e^{-(r-\gamma)D} - D(r-\gamma)e^{-(r-\gamma)D} - T^*(r-\gamma)(1 - e^{-(r-\gamma)D})\right]}{(r-\gamma)\gamma[1 - e^{-(r-\gamma)T^*} + \alpha(1 - e^{-(r-\gamma)D})]}$$

As $1-e^{-x}-x<0$ and $\left[1-e^{-(r-\gamma)D} - D(r-\gamma)e^{-(r-\gamma)D} - T^*(r-\gamma)(1 - e^{-(r-\gamma)D})\right]$ is a decreasing function of T^* and is zero for $T^* = 0$, we obtain a negative relation between the growth rate and the lifetime of machines.

(ii) We directly obtain the effect of the parameters analyzed on H; by differentiating equation (7) taking into account equation (6):

$$\frac{\partial H}{\partial D} = -\frac{\left(\frac{\partial T^*}{\partial D} + \alpha\right)}{H^2} < 0$$

$$\frac{\partial H}{\partial \alpha} = -\frac{\left(\frac{\partial T^*}{\partial \alpha} + T\right)}{H^2} < 0$$

$$\frac{\partial H}{\partial \gamma} = -\frac{1}{H^2}\frac{\partial T^*}{\partial \gamma} > 0$$

(iii) We first check that adoption costs affect detrended output in a negative way. Replacing H in equation (10), taking into account equation (9), and differentiating with respect to D gives:

$$Y = \frac{1 - e^{-\gamma T}}{\gamma(T + \alpha D)} \tag{29}$$

$$\frac{\partial Y}{\partial D} = \frac{1}{\gamma(T^* + \alpha D)}\left[\left(\gamma e^{-\gamma T^*}(T^* + \alpha D) - 1 + e^{-\gamma T^*}\right)\frac{\partial T^*}{\partial D} - \alpha(1 - e^{-\gamma T^*})\right]$$

The result is obvious as $\frac{\partial T^*}{\partial D} > 0$, and the function $\left[\gamma e^{-\gamma T^*}(T^* + \alpha D) - 1 + e^{-\gamma T^*}\right]$ takes the value 0 for $T^* = 0$ and decreases for $T^* > 0$. Differentiating (26) with respect to α we obtain the same positive effect.

To check the inverse relation between detrended output and technological progress, we define $x = \gamma T^*$, so

$$Y = \frac{1 - e^{-\gamma T^*}}{\gamma T^* + \alpha\gamma D} = \frac{1 - e^{-x}}{x + \alpha\gamma D} = G(x(\gamma); \gamma)$$

and

$$\frac{\partial Y}{\partial \gamma} = \frac{\partial G}{\partial x}\frac{\partial x}{\partial \gamma} + \frac{\partial G}{\partial \gamma}$$

Differentiating G with respect to γ, and x, we obtain an inverse relation in both cases:

$$\frac{\partial G}{\partial \gamma} = -\frac{\alpha D(1 - e^{-x})}{(x + \alpha\gamma D)^2} < 0$$

$$\frac{\partial G}{\partial x} = \frac{e^{-x} + xe^{-x} - 1 + \alpha\gamma D e^{-x}}{(x + \alpha\gamma D)^2} < 0$$

To check the negative relation between the rate of technical progress and the optimal lifetime, we have to prove that $\frac{\partial x}{\partial \gamma} > 0$; that is, γT^* is increasing with respect to γ

$$\frac{\partial(\gamma T^*)}{\partial \gamma} = T^* + \gamma\frac{\partial T^*}{\partial \gamma} = T^* - \gamma\frac{\frac{\partial F}{\partial \gamma}}{\frac{\partial F}{\partial T^*}}$$

where

$$F = \frac{1 - e^{-rT^*}}{r} - \frac{e^{-\gamma T^*} - e^{-rT^*}}{r - \gamma} - \frac{\alpha e^{-\gamma T^*}(1 - e^{-(r-\gamma)D})}{r - \gamma} - 1 = 0$$

$$\frac{\partial F}{\partial T^*}\frac{\partial(\nu T^*)}{\partial \gamma} = T^*\frac{\partial F}{\partial T^*} - \gamma\frac{\partial F}{\partial \gamma}$$

and we have already shown that $\frac{\partial F}{\partial T^*} > 0$ and $\frac{\partial F}{\partial \gamma} > 0$; thus $\frac{\partial(\nu T^*)}{\partial \gamma} > 0$

Section 7. Comparative Statistics Results.

Jacobian Matrix of the system $[(F);(G)] \equiv (S)$

$$(F) \qquad \frac{1-e^{-rT}}{r} - \frac{e^{-\nu T}-e^{-rT}}{r-\gamma} - \frac{\alpha e^{-\nu T}(1-e^{-(r-\gamma)D})}{r-\gamma} - 1 = 0$$

$$(G) \qquad \frac{1-e^{-rT}}{r} + \frac{\alpha'(K)e^{-\nu T}(1-e^{-(r-\gamma)D})}{r-\gamma} - 1 = 0$$

The Jacobian Matrix of the system can be expressed by:

$$J = \begin{vmatrix} F_T & F_K \\ G_T & G_K \end{vmatrix}$$

Where the first and the second columns of J refer to the partial derivatives of the LHS of (S) respectively w.r.t T and K, and where F_T; F_K; and G_T are strictly positive, and G_K is strictly negative.

The Jacobian determinant det J is thus strictly negative:

$$\det J = F_T G_K - G_T F_K < 0$$

We denote F_{α_0}, G_{α_0}, F_D, G_D, F_γ, G_γ the partial derivatives of S w.r.t α_0, D; and γ, where F_{α_0}, F_D, and G_D are strictly negative, F_γ, G_γ, F_γ, G_γ are strictly positive and G_{α_0} is equal to zero. By Cramer's rule we obtain:

$\frac{\partial T}{\partial D} > 0$; $\frac{\partial T}{\partial \alpha_0} > 0$; $\frac{\partial T}{\partial \gamma} < 0$; $\frac{\partial K}{\partial \alpha_0} > 0$, and an ambiguous effect for $\frac{\partial K}{\partial D}$ and $\frac{\partial K}{\partial \gamma}$

Short Run Dynamics of Job Creation

In all the figures we set $r = 0.05$, $\gamma = 0.03$, and we vary D, and α to isolate the effect of each one of them in the dynamics of $H(t)$. In figures 3-5, the initial conditions are supposed to be increasing: $H_0(t) = ae^{0.02t}$ for $t < 0$. In contrast, we assumed cyclical initial conditions in figures 6-8; for the previous figures we set the following initial conditions,

$H_0(t) = b + \frac{1}{10T^*}\sin(\frac{\pi t}{T^*})$

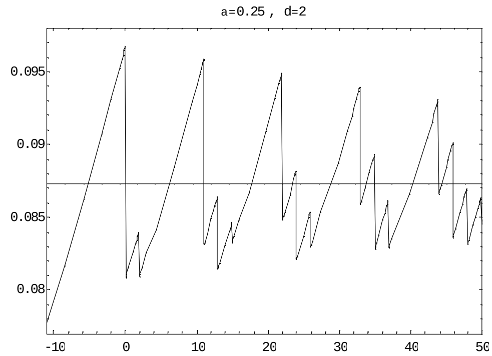

Figure 3.

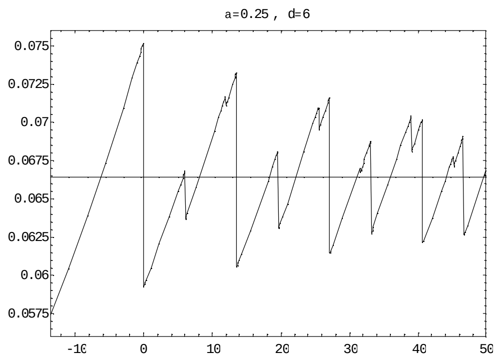

Figure 4.

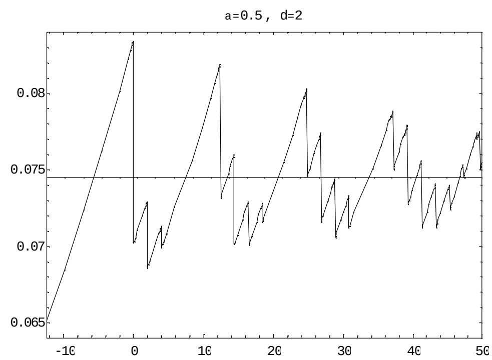

Figure 5.

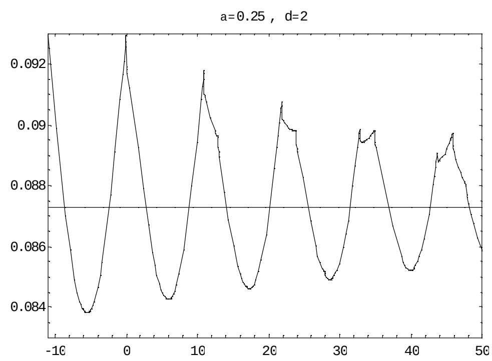

Figure 6.

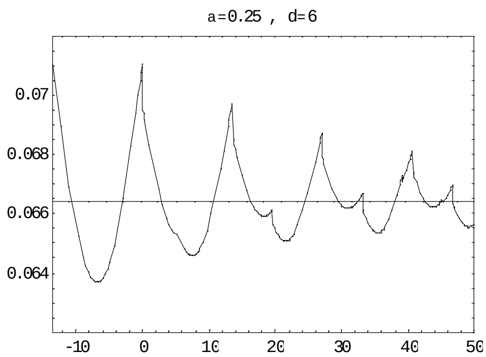

Figure 7.

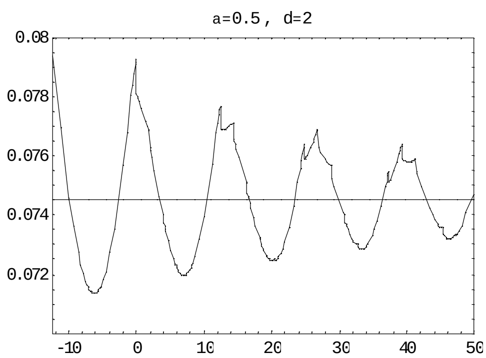

Figure 8.