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Other-Regarding Preferences in Organizational Hierarchies

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Other-Regarding Preferences in Organizational Hierarchies

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Abstract

In this paper, we provide new theoretical insights about the role of collusion in organizational hierarchies by combining the standard principal-supervisor-agent framework with a theory of social preferences. Extending Tirole's (1986) model of hierarchy with the inclusion of Fehr and Schmidt's (1999) distributional other-regarding preferences approach, the links between inequity aversion, collusive behavior throughout the levels of a hierarchy and the changes in optimal contracts are studied. It turns out that other-regarding preferences do change the collusive behavior among parties depending on the nature of both the agent's and the supervisor's other-regarding preferences. Most prominent impact is on the optimal effort levels. When the agent is inequity averse principal can exploit this fact to make agent exert higher effort level than she would otherwise. In order to satisfy the participation constraint of the supervisor, the effort level induced for the agent becomes lower when the supervisor is status seeker, and it is higher when the supervisor is inequity averse.

Keywords: Other-Regarding Preferences, Hierarchy, Collusive Behavior, Optimal Contract Design

JEL Codes: D90, D82, L22

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1 Introduction

Many models in economics use self-interest approach for the analysis of real life problems. However, countless lab and field experiments in economics showed that self-interest approach may not be sufficient in explaining all of the observed behavior. (See Ledyard (1995), and Fehr & Schmidt (2006) for evidences on public and private good environments respectively.) Moreover, these experiments highlighted the fact that many people show other-regarding preferences, and concern for another's situation is a motivating factor in most people's decision making processes. Supporting the experiments, many theoretical papers showed that this behavior can be presented in a tractable way. We also follow this path and look for how other-regarding preferences shape principal-supervisor-agent hierarchy in an organization. We particularly investigate the impact of other-regarding preferences on the collusive behavior in a hierarchy and on the optimal contract design. We show that collusion between the hierarchy levels and wage payments in an optimal contract depend on the degree of social preferences. Moreover, the role of collusion can be ignored in an organization if a supervisor's level of inequity aversion is above a certain threshold.

Collusive behavior is not a rare event in organizations. The fact that several layers of people interact with each other puts an emphasis on group gains as well as individual gains which opens a way to forming coalitions among different parties. Hence, hierarchy design for an organization must consider the possibility of corruptive activities and aim to prevent them beforehand. To present a way to construct this kind of design, in his seminal work, Tirole (1986) puts sociological studies on corruption in hierarchies into a formal theoretical model by adding a supervisor layer between the standard principal-agent network. However, although he acknowledges the fact that interaction between supervisor and agent affects the shape of their relationship he prefers to use self-interested parties in his theoretical framework.

The traditional literature in standard principal-agent contract design also uses the assumption of self-interested behavior. However, several papers on principal/agent relationships have started to consider the theories of social preferences in their models. (See, among others, Itoh (2004), Rey-Biel (2008), and Neilson & Stowe (2010)) On the other hand, such models have not been widely affiliated with the collusive behavior and optimal contracts in a basic three level hierarchy, though Tirole (1986), for example, considers this type of hierarchies as a network of standard two-tier contracts.

We provide a theoretical framework to complement empirical studies on how social preferences in a hierarchy influence collusive tendency and behavior of the parties and hence optimal contract design. Several empirical studies (Agell & Lundborg (1995), Bewley (2002), Blinder & Choi (1990), and Campbell III & Kamlani (1997)) report that employees in an organization not only care about their own well-being but also take the well-being of their co-workers into consideration, and managers designs contracts that avoids too much internal inequality. Pepper & Gore (2015) argues that an employee of an organization may take the compensation (rewards) of her peers, immediate subordinates or immediate superiors as a reference for her own rewards and compensation. Hence, it appears essential to apply other-regarding preferences into hierarchical models. By incorporating other-regarding preferences, it is possible to gain further and more realistic insights about collusive behavior and design of optimal contracts.

The first objective of this paper is to introduce other-regarding preferences for the supervisor and agent in an organization and investigate its effect on the collusive behavior. The second one is to explore the changes in the optimal contract parameters, effort levels exerted by the agent, and the supervisor's and agent's wage structure at different states. We choose to use Fehr & Schmidt (1999) type of other-regarding utility function, since it is simple, powerful and can easily be applied to the three-layer principal-supervisor-agent hierarchy. We investigate the case in which supervisor is either inequity averse or status seeker, and agent is only inequity averse since we assume that the agent's wage is always lower than the wage of the supervisor.

The main observation is that introducing other-regarding preferences changes the amount that the briber has to pay in order to persuade the bribed party and also the amount that she can pay at most if she wants to form a coalition. The components of contracts (especially efforts exerted by the agent) also change with other-regarding preferences. Though the ranking of agent's and supervisor's utilities at different states does not differ from Tirole's (1986) model, wages and dispersion between the wages at different states vary due to the changes in collusion constraints shaped by social preferences. Furthermore, we find that inequity aversion results in wage contraction between the layers of the hierarchy. Lastly, depending on the degree of social preferences the principal can escape from the burden of preventing collusive behavior, since the net benefit of the collusion can be negative for agents with other-regarding preferences.

The next section constructs and explains our main model, which is basically Tirole's (1986) model with Fehr and Schmidt (1999) type other-regarding preferences. Afterwards, we analyze the principal's problem and investigate the structure of optimal contracts with collusion when the supervisor and the agent have other-regarding preferences and provide the main results. We conclude the paper by discussing main points and results, and state the remaining open questions for future research. Proofs of the main results are relegated to the Appendix.

2 The Model

Our model is built on Tirole's (1986) three level (principal-supervisor-agent) hierarchy model. In order to show the effects of inequity aversion on the collusive behaviour and optimal contracts, we use Fehr and Schmidt's (1999) theory of inequity aversion in the definition of supervisor's and agents' utilities.[1]

The Parties: The only productive unit in this hierarchy is the agent. Principal earns profit from the agent's productive effort $e > 0$ and the productivity parameter θ in the

[1] We refer the reader to Tirole's and Fehr and Schmidt's papers for more about the discussion and justification of the environment and assumptions.

environment according to the technology $x = \theta + e$.

Agent faces disutility coming from the exerted effort level, and it is shown as $g(e)$ in monetary terms where g is strictly convex, increasing in effort, and $g(0) = g'(0) = 0$. Principal pays a wage W to the agent.

Supervisor's main and the most important role in this hierarchy is to monitor the agent and her environment, and then report the result of her inspection about productivity θ to the principal. The details about supervisor's duty is explained later with the definition of hidden action and hidden information problem existing in our model. As in Tirole (1986), we impose a simple supervision technology in which supervisor exerts no effort while observing the agent's productivity; thus, there is no monitoring cost for the supervisor. Principal pays a wage S to the supervisor.

The assumption about the supervisor's and agent's utility functions is the main difference between our model and Tirole's (1986) model, and this is our main contribution. Inequity aversion approach suggests that interacting participants in hierarchical environments do care about not only their own payoffs but also the payoff of the interacted party. In addition, other-regarding preferences occur among people with similar social circles according to experiments. (See, among others, Agell and Lundborg (1995), Bewley (2002), Blinder and Choi (1990), Campbell and Kamlani (1997), and Pepper and Gore (2015)) Although the supervisor and agent are in different levels of hierarchy it is reasonable to assume that supervisor and agent see each other as co-workers; hence, we assume that other-regarding behaviour is observed between supervisor and agent in the principal-supervisor-agent hierarchy. Since principal is in the owner role of the whole game her total gain is not considered in the utilities of supervisor and agent, or the payoffs of the supervisor and agent does not have any effect on the utility of principal.

Agent's utility in our environment involves W and $g(e)$. Thus, one should has to decide on whether comparison between payoffs of interacting parties includes both pay and effort cost of the agent or only the wage paid W. Neilson and Stowe (2010) argues that interacting parties can only observe the wages of his co-workers (when wages are observable) and they probably tend to not consider the effort differences - which is also a source of payoff differences- among themselves. Moreover, since a principal can not observe the effort levels of agents perfectly (hidden action problem) it is reasonable to assume that workers may not observe their co-workers' effort levels perfectly. As a result, to include disutility coming from effort levels in the comparison between payoffs seems to be a difficult task. Similar action has been taken by Itoh (2004) in one of his models, where the agent and principal only care about the other party's payoff in their other-regarding preferences. Hence, we assume that the supervisor and agent only consider wage levels and ignore the cost of effort levels in the comparison of their utilities.

In the specifications of the utilities of the supervisor and the agent, we also assume that the party at the supervisor level earns a higher wage in every state of information, i.e, $S_i > W_i$.[2] This assumption corresponds to the most of wage settings in the real world hierarchies, and it also simplifies the construction and solution of our model without weakening its applicability. Thus, this assumption can be considered as a natural specification in a hierarchical inequity aversion setting.

Using Fehr and Schmidt's (1999) two player utility function with inequity aversion, we define the utility of the agent as:

$$U(W - g(e) - \lambda_A(S - W)),$$

and the supervisor's utility as:

$$V(S - \lambda_S(S - W)),$$

where λ_A and λ_S represents inequity aversion parameters of the agent and the supervisor respectively. Since $S > W$ at all states, $\lambda_A > 0$ (agent is inequity averse and dislikes being behind) and $\lambda_S < 1$ (either supervisor is inequity averse as well as behindness averse,

[2] Index i denotes the states of nature.

$0 < \lambda_S < 1$, or supervisor loves being ahead and she is status-seeker (competitive), $\lambda_S < 0$). As λ_A increases the agent becomes more inequity averse, i.e, becomes more sensitive to being behind. For $0 < \lambda_S < 1$, increase in λ_S makes the supervisor more inequity averse and more sensitive to being ahead. On the other hand, for $\lambda_S < 0$, as λ_S decreases the supervisor becomes more status-seeker and thrives more from being ahead. The case where $\lambda_A = 0$ and $\lambda_S = 0$ represents our benchmark case, Tirole (1986). Both U and V are differentiable, strictly concave and increasing Von Neumann Morgenstern utility functions with $U'(0) = \infty$ and $V'(0) = \infty$. We use expected utilities for both the supervisor and the agent in the analysis of our model since there are different states of nature. The expected utility of the agent is $EU(W - g(e) - \lambda_A(S - W))$ and the expected utility of the supervisor is $EV(S - \lambda_S(S - W))$.

The supply of supervisors and agents has competitive nature, and agents have reservation wages W_0 with reservation utility $\overline{U} \equiv U(W_0)$, while supervisors have reservation wages S_0 with reservation utility $\overline{V} \equiv V(S_0)$. The participation (individual rationality) constraints for the agent and supervisor can be respectively written as:

$$EU(W - g(e) - \lambda_A(S - W)) \geq \overline{U},$$

$$EV(S - \lambda_S(S - W)) \geq \overline{V}.$$

The last party of this organizational hierarchy is the principal. She assigns agents to the work project, and designs and offers contracts to both supervisor and agent. The assumption about the principal is that she is risk-neutral. Her expected utility is defined as:

$$E(x - S - W) = E(\theta + e - S - W)$$

Hidden Information Problem: There are two levels of productivities in the working environment, low state of productivity $\underline{\theta}$ and high state of productivity $\overline{\theta}$, where $0 < \underline{\theta} < \overline{\theta}$, and $\Delta\theta = \overline{\theta} - \underline{\theta}$. $\underline{\theta}$, $\overline{\theta}$ and their probability of occurrence are common knowledge.

The agent always becomes aware of the productivity level in the environment after signing the contract, and determines her effort level according to productivity realized. However, the supervisor sometimes fails to observe the productivity level in the environment. Thus, when supervisor monitors the agent one of the four following states of nature (indexed by i) can arise:

State 1: Both agent and supervisor observe low level of productivity $\underline{\theta}$.

State 2: Agent observes $\underline{\theta}$. However, supervisor fails to observe current productivity level.

State 3: Agent observes high level of productivity $\overline{\theta}$. However, supervisor fails to observe current productivity level.

State 4: Both agent and supervisor observe $\theta = \overline{\theta}$.

Each state of nature has a probability of occurrence p_i where $\sum_{i=1}^{i=4} p_i = 1$.

Lastly, it is assumed that the agent has information on whether the supervisor observed the productivity level successfully or not. However, the agent can not report supervisor's monitoring activity to the principal. The information structure becomes poorer as we go through the upper levels in the hierarchy. Moreover, the supervisor, as well as the principal, can not observe the level of effort exerted by the agent.

Timing: First, the principal offers contracts to both parties. S and W are specified in this contract as functions of observable contractible variables which are the output, x, and the report of the supervisor on the current productivity level, r, and supervisor's and agent's inequity aversion parameters, λ_S and λ_A, which are assumed to be common knowledge for all parties in the organization. Both supervisor's and agent's wages become common knowledge to all parties when contracts are offered.

If contracts are accepted and before the uncertainty is resolved, i.e. parties do not know the level of θ, the supervisor and agent move to a collusion stage and try to decide on the side transfers between both parties.[3] Similar to the main contract offered by the principal,

[3] We refer the reader to Tirole's (1986) paper for detailed information on possible state misrepresentations and conditions for signing a side contract between a supervisor and an agent.

a side transfer is also a function of output, x, supervisor's report on productivity level, r, and inequity aversion parameters of the colluding parties, λ_A, and λ_S. Side transfers are not observable by the principal.

In the next step, the agent learns productivity level in the environment, and then agent chooses her effort level. It means that the profit is also realized. On the other hand, supervisor may or may not observe the productivity level. She prepares her report and presents it to the principal. If she fails to observe θ, her report is considered empty, $r = \phi$. In the case where the supervisor observes the productivity level successfully, she has the option to report her monitoring in a truthful manner or to hide the true information and give empty report, i.e. the supervisor's report is $r \, \epsilon \, \{\theta, \phi\}$.[4] One of the four states, i.e. the outcome of the monitoring and collusion game, is realized after the reports.

The final step is the execution of contracts. The principal pays S and W to the supervisor and the agent respectively after she learns the output level and report of the supervisor. Moreover, if the supervisor and agent decides to form a coalition at the collusion stage, side transfers are allocated.

First Best Solution with Self-Interested Parties (Tirole (1986)): In order to use the results as a reference point later, consider the case when principal can observe productivity levels in the environment perfectly (no hidden information problem), and the effort level exerted by the agent (no hidden action problem). Moreover, all of the parties in hierarchy is self interested utility maximizers ($\lambda_A = 0$, $\lambda_S = 0$).

In this case, principal does not need a supervisory duty. We can find optimal effort level e^* for the agent by solving the following optimization problem:

$$\max_{e} \{\theta + e - W_0 - g(e)\}.$$

[4]Throughout the discussion on the context of supervisor's report, we assume that when the supervisor observes the level of productivity in the environment her report is considered as credible by the principal. On the other hand, the agent can not make verifiable and credible announcements about the productivity level.

It is easy to see that $g'(e^*) = 1$ for both $\underline{\theta}$ and $\overline{\theta}$. Moreover, the wage paid to the agent is $W = W_0 + g(e^*)$ in every state of nature.

3 Analysis of the Principal's Problem

The rest of our analysis follows the methodology below:

1) All of the post-side-transfer allocation constraints (individual rationality constraints, incentive compatibility constraints, and no collusion constraints) are introduced to the principal's problem of optimal contract design.

2) Principal's net expected profit is maximized subject to given constraints.

3) Results are analyzed and compared with the benchmark case where agents are self interested, i.e. $\lambda_A = 0$, $\lambda_S = 0$, (Tirole, 1986).

First, note that the wage of the supervisor is always higher than the agent's wage since the former is at the upper level of the hierarchy ($S_i > W_i$). As a result, the agent can only be inequity averse ($\lambda_A > 0$). On the other hand, the supervisor can show two different behaviors. She may be either inequality averse and feels bad about the fact that she is earning a higher wage than her co-worker, or status-seeker and thrives from being ahead which is a sign of her position (status) against her co-worker. For the status-seeker supervisor $\lambda_S < 0$, and $1 > \lambda_S > 0$ is used for the inequity averse one.

We can now define the constraints for the principal's optimal contract design problem. The participation constraints for supervisor and agent must be satisfied so that the main contract is accepted by both parties at the first place. The participation (individual rationality) constraints, for the supervisor and the agent respectively, are defined as follows:

$$(SPC): \quad EV(S - \lambda_S(S - W)) = \sum_i p_i V(S_i - \lambda_S(S_i - W_i)) \geq \overline{V} \equiv V(S_0)$$

$$(APC): \quad EU(W - g(e) - \lambda_A(S - W)) \sum_i p_i U(W_i - g(e_i) - \lambda_A(S_i - W_i)) \geq \overline{U} \equiv U(W_0)$$

There is a hidden action problem in our model since the principal and supervisor can not observe the effort level exerted by the agent. In states 1 and 4, principal has the knowledge on productivity levels. Hence, she can deduct the effort level of the agent by looking at the output. However, this is not the case for states 2 and 3. When state 3 is realized agent can claim that it is actually state 2, and the profit of the principal is achieved on low level of productivity $\underline{\theta}$ with the hard work of the agent, though true state of nature is actually $\overline{\theta}$. With her false information, she is able to exert less effort $e_2 - \Delta\theta$ instead of e_2 but earn the wage W_2 as if she exerts e_2. The principal must provide necessary incentives to make agent exert higher level of effort when supervisor's monitoring fails. The incentive compatibility for the agent is defined as follows:

$$(AIC): \quad W_3 - g(e_3) - \lambda_A(S_3 - W_3) \geq W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2)$$

In order to cease coalition between the supervisor and agent, the principal must arrange the payoffs of both parties such that the total amount that the briber has to pay for a potential collusion must not be lower than how much she can actually pay. The principal tries to increase the amount of minimum side transfer at which the bribed party is indifferent between accepting and rejecting the bribe offer, and/or decrease the maximum amount of side payment the briber can provide without making herself worse off than in the alternative case where she stays loyal to the true nature of state. Collusion constraints can be seen as a formalization of these two strategies.

For a coalition hiding the low productivity in state 1, maximum side transfer must satisfy $W_2 - t_{max,1} - g(e_2) - \lambda_A(S_2 + t_{max,1} - W_2 + t_{max,1}) = W_1 - g(e_1) - \lambda_A(S_1 - W_1)$ such that it is not rational for the agent to offer more than $t_{max,1}$ when she wants to make supervisor provide false report about productivity. Moreover, the agent has to transfer at least $t_{min,1}$, satisfying $S_2 + t_{min,1} - \lambda_S(S_2 + t_{min,1} - W_2 + t_{min,1}) = S_1 - \lambda_S(S_1 - W_1)$, to the supervisor so that the latter is indifferent to the bribe. Then, in order to prevent collusion, the principal must arrange the main contract such that $t_{min,1} \geq t_{max,1}$ (CIC1, first collusion constraint).

For the second collusion possibility, hiding the truth about high productivity environment at state 4, maximum side transfer paid by the agent must satisfy $W_3 - t_{max,2} - g(e_3) - \lambda_A(S_3 + t_{max,2} - W_3 + t_{max,2}) = W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. In this case, the supervisor gets at least $t_{min,2}$, satisfying $S_3 + t_{min,2} - \lambda_S(S_3 + t_{min,2} - W_3 + t_{min,2}) = S_4 - \lambda_S(S_4 - W_4)$, to accept the side transfer. Then, the principal must arrange the main contract such that $t_{min,2} \geq t_{max,2}$ (CIC2, second collusion constraint).

It is easy to see that it is impossible for the agent to bribe the successful supervisor when $\lambda_S \to 0.5$. The collusion constraints representing these cases $(CIC1)$ and $(CIC2)$ are dropped from the principal's problem. When the supervisor's sensitivity to wage inequality is $\lambda_S = 0.5$, she does not accept any coalition offer coming from the agent. It is because of the fact that there exists a certain value for λ_S where the disutility coming from accepting the bribe and increasing the inequality offsets the monetary gain of the corresponding side transfer completely. Moreover, for the values above this threshold ($\lambda_S > 0.5$), disutility from increasing inequality dominates the side payment's monetary payoff when the agent comes with a bribe offer. This implies that the role of collusion in organizational hierarchies can be ignored if a supervisor's level of inequity aversion is above the certain threshold which is 0.5 in our environment. Therefore, we assume that $0 < \lambda_S < 0.5$ for the rest of the paper. Given this observation, we can now define the first two collusion constraints:

$$(CIC1): \quad \frac{S_1 - \lambda_S(S_1 - W_1)}{1 - 2\lambda_S} + \frac{W_1 - g(e_1) - \lambda_A(S_1 - W_1)}{1 + 2\lambda_A} \geq \frac{S_2 - \lambda_S(S_2 - W_2)}{1 - 2\lambda_S} + \frac{W_2 - g(e_2) - \lambda_A(S_2 - W_2)}{1 + 2\lambda_A},$$

$$(CIC2): \quad \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} \geq \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A}.$$

Note that, for the collisions at states 1 and 4, paying a side transfer not only decrease the agent's monetary payoff but also make her feel worse by increasing the inequality between her and the supervisor. Therefore, maximum side transfer the agent can provide is reduced for an inequity averse agent in comparison to a self-interested one. On the other hand, paying a side transfer to the status-seeker supervisor not only increase her monetary payoff directly but also make her feel better by increasing the wage difference between her and the agent, i.e. by increasing her position (status) against the agent. Hence, a side transfer may make supervisor more prone to accept a bribe offer in comparison to the benchmark case. Things are different for an inequity averse supervisor. Accepting a side transfer make her feel worse due to the increase in inequality between her and the agent, and negates the positive effect on the payoff coming from the monetary side payment. Thus, a side transfer's impact on making the supervisor accept a bribe offer is reduced in comparison with the benchmark case.

Another coalition may occur at state 3 (a failure of supervisor's monitoring) when the supervisor wants to bribe the agent to imitate an agent with low productivity environment and accept the incentive scheme of state 2. In this case, minimum side transfer must satisfy $W_2 + t_{min,3} - g(e_2 - \Delta\theta) - \lambda_A(S_2 - t_{min,3} - W_2 - t_{min,3}) = W_3 - g(e_3) - \lambda_A(S_3 - W_3)$ so that the agent falsely claims that it is state 2. Moreover, maximum side transfer must satisfy $S_2 - t_{max,3} - \lambda_S(S_2 - t_{max,3} - W_2 - t_{max,3}) = S_3 - \lambda_S(S_3 - W_3)$ such that it is not rational for the supervisor to offer more than $t_{max,3}$ at most without making herself worse off than no coalition case. Hence, in order to prevent collusion, the optimal contract must be arranged such that such that $t_{min,3} \geq t_{max,3}$ (CIC3, third collusion constraint). That is,

$$(CIC3): \quad \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} \geq \frac{S_2 - \lambda_S(S_2 - W_2)}{1 - 2\lambda_S} + \frac{W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2)}{1 + 2\lambda_A}$$

It is important to note the fact that paying a side transfer to an inequity averse agent not only increase her monetary payoff but also make her feel better by decreasing the inequality between her and the supervisor. Thus, in comparison to self-interested agent, a smaller side transfer may make the inequity averse agent accept a bribe offer. On the other hand, in this case, paying a side transfer to the agent not only decrease the monetary payoff of the status-seeker supervisor but also make her feel worse due to disutility coming from the reduction in her relative status against the agent. Thus, the maximum amount of side transfer the supervisor can provide is reduced for the status-seeker supervisor in comparison to the benchmark. However, a side transfer to an agent make an inequity averse supervisor feel better due to the decrease in inequality, and dampens the negative effect of monetary loss. Hence, the maximum amount of the side transfer is increased for the inequity averse supervisor in comparison with the benchmark.

We now solve the optimal contract design problem for the principal. By choosing the optimal S_i, W_i, e_i values, the principal wants to maximize her expected utility, $\sum_i p_i(\theta_i + e_i - S_i - W_i)$, subject to constraints introduced. In other words, we have to solve the following problem:

$$\max_{(S_i, W_i, e_i)} \sum_i p_i(\theta_i + e_i - S_i - W_i)$$

subject to

$$(SPC), (APC), (AIC), (CIC1), (CIC2), and\ (CIC3)$$

In order to expose the impact of the other-regarding preferences, we first present the benchmark results by setting λ_A and λ_S equal to zero. This case corresponds to the problem tackled by Tirole (1986).

Theorem 1 ***(Tirole (1986))*** *The solution to principal's problem with the self interested supervisor and agent (benchmark case) has the following properties:*

a) $S_4^B > S_1^B > S_2^B = S_3^B$

b) $W_3^B - g(e_3^B) > W_4^B - g(e_4^B) > W_1^B - g(e_1^B) > W_2^B - g(e_2^B)$ *and* $W_3^B > W_4^B > W_1^B > W_2^B$

c) $S_4^B + W_4^B = S_3^B + W_3^B$

d) $e_1^B = e_3^B = e_4^B = e^* > e_2^B$

e) All the constraints in the benchmark problem, except $(CIC1^B)$*, are binding.*[5]

[5]The superscript B denotes the benchmark values for our problem.

The following result shows the effects of other-regarding preferences on the design of optimal collusion-proof contract.

Proposition 1 The solution for the effort levels to the principal's problem with the other-regarding agent and supervisor has the following properties:

a) $e_1 = e_3 = e_4 = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} > e_2$

b) $\frac{\partial e_1}{\partial \lambda_A} = \frac{\partial e_2}{\partial \lambda_A} = \frac{\partial e_3}{\partial \lambda_A} = \frac{\partial e_4}{\partial \lambda_A} > 0$ and $\frac{\partial e_1}{\partial \lambda_S} = \frac{\partial e_2}{\partial \lambda_S} = \frac{\partial e_3}{\partial \lambda_S} = \frac{\partial e_4}{\partial \lambda_S} > 0$

c) When $\lambda_A > -\lambda_S$, i.e. inequity aversion behavior of the agent dominates the status seeking of the supervisor: $e_1 = e_3 = e_4 > e^*$ and $e_2 > e_2^B$, there are threshold $\lambda_A(\Delta\theta, \lambda_S)$ and $\lambda_S(\Delta\theta, \lambda_A)$ values at which $e_2 = e^*$.

When $\lambda_A < -\lambda_S$, i.e. status seeking behavior of the supervisor dominates the inequity aversion of the agent: $e_2 < e_1 = e_3 = e_4 < e^*$ and $e_2 < e_2^B$.

When $\lambda_A = -\lambda_S$: $e_1 = e_3 = e_4 = e^* > e_2$ and $e_2 = e_2^B$.

Proposition 1.a. states that the effort level induced by the principal is lower at state 2 when productivity is low, $\underline{\theta}$, and supervisor's report is empty, $r = \phi$. This result is not because of the measures taken to prevent collusive behavior but to provide necessary incentive to the agent- in order to satisfy (AIC) - which makes her stay at the the true productivity level $\overline{\theta}$ at state 3. When supervisor fails to monitor the agent, the principal induces lower effort at state 2 in comparison to state 3 so that she can make a reduction on W_2. The attractiveness of imitating the low productivity state is diminished.

The first inequality in Proposition 1.b. implies that as the agent cares more about the wage inequality, the effort level induced by the principal increases at every state. This is due to the fact that the inequity averse agent does not enjoy being behind and her wage increases with her effort level. The principal can exploit this fact and moderates the increasing cost of effort with the decrease in wage inequality. As agent becomes more sensitive to being behind, it is easier to offset the cost of extra effort which means that the principal can induce higher and higher effort. Therefore, in terms of effort levels, the principal is more likely to choose an agent with higher inequity aversion sensitivity since higher level of effort means higher level of output for the principal.

The second part of Proposition 1.b. specifies two distinct cases. First of all, inequity averse supervisor does not enjoy being ahead, and the principal increases wage of the agent in order to reduce wage inequality and satisfy the supervisor's participation constraint. A rise in the wages persuades the agent to exert a higher level effort. The increase in the agent's wage is somewhat compensated with a higher effort level and higher level of output. Furthermore, as the supervisor becomes more inequity averse the increase in agent wages must be higher. This means that the principal should induce higher effort levels.

Note that above the optimal effort level, the cost of effort which induces the same profit increase as the wage increase is higher than the utility gain of the agent coming from a rise in her wage. Since the principal has to satisfy the agent's participation constraint at the same time, having an inequity averse supervisor in the hierarchy may hurt the principal, and it becomes more disadvantageous as the the supervisor becomes more sensitive to wage inequality. Even if this is the case, it is always better than increasing supervisor's wages solely to compensate the disutility coming from the wage inequality since supervisor is not a productive unit in our hierarchy model.

Additionally, the status-seeker supervisor enjoys being ahead since she sees the wage inequality as a sign of status. As a result, principal decreases the agent's wage to use supervisor's personality and exploits wage inequality. Reduction in wages means that the agent now should exert lower level of effort than before. As the supervisor becomes more sensitive to her sign of status, the agent's wage and consequently the effort induced fall further. It is not unreasonable to think that inducing less effort in this case can be seen as a lost opportunity. However, we want to remark that the supervisor earns a wage without producing anything and lowering agent's wages also opens a path in which the principal can also reduce supervisor's wage thanks to the latter's enjoyment from wage inequality (note also that the supervisor always gets higher wage since she is at the upper level in the hierarchy).

Proposition 1.c. is directly related to Proposition 1.b. because self interested benchmark agent and supervisor have zero sensitivity against wage inequality. When both the agent and supervisor are inequity averse, principal can induce higher effort levels at every state in comparison to the benchmark case. Moreover, in case where the hierarchy has a competitive supervisor, if the inequity aversion sensitivity of the agent is higher than the responsiveness of supervisor to her higher status, the need for extra wage (extra effort) to reduce inequality has priority over taking advantage of status seeking behavior of the supervisor, and the reduction of effort induced. As a result, we observe higher levels of effort than the benchmark at every state again. In all of these cases, the principal makes agent exert more than e^* at states 1, 3 and 4. If the cost moderation effect of reducing wage inequality combined with the necessary change in agent wages to satisfy the other-regarding supervisor's participation constraint dominates the need for inducing lower effort and - consequently- lower wage at state 2 to satisfy (AIC), the principal can also induce more than the benchmark effort level at state 2. Since higher the difference between high and low productivity levels are lower the effort level induced at state 2, the principal must employ an agent and/or a supervisor with higher inequity aversion sensitivity if she wants to induce more than the benchmark effort level at state 2.

In case where the competitiveness of the supervisor dominates the inequity aversion of the agent, the need for extra wage (extra effort) to reduce inequality is inferior to exploitation of status seeking behavior of the supervisor, and the reduction of effort induced. This time, the principal has to induce less than e^* at all states and all of the effort levels are lower than their benchmark counterparts.

Lastly, the need for decrease in the agent's wages and efforts in order to benefit from exploiting competitive behavior of the supervisor offsets the necessity of extra wage and effort to reduce wage inequality if the status seeking intensity of the supervisor and inequity aversion sensitivity of the agent are equal to each other. In that case, we observe exactly the same results that we see in the benchmark case.

Proposition 2 All of the constraints, except $CIC1$, introduced to the principal's problem with other-regarding agent and supervisor have positive shadow prices, i.e. they are binding.

This is exactly the same result we see in the benchmark case, since whether the parties are inequity averse, status seeker or self interested the agent always prefer to let it known that the reason of lower level of output at state 2 is the low productivity environment. In that case, the supervisor also supports the agent and acts like an advocate for her by reporting the true state of nature.

Proposition 3 The optimal wages with the other-regarding agent and supervisor has the following properties:

a) $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_3 - \lambda_S(S_3 - W_3) = S_2 - \lambda_S(S_2 - W_2)$

b) $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$

c) $\frac{S_4-\lambda_S(S_4-W_4)}{1-2\lambda_S} + \frac{W_4-\lambda_A(S_4-W_4)}{1+2\lambda_A} = \frac{S_3-\lambda_S(S_3-W_3)}{1-2\lambda_S} + \frac{W_3-\lambda_A(S_3-W_3)}{1+2\lambda_A}$

Proposition **??**.a. shows the ranking of supervisor's possible utility levels at different states. The ranking is same with the benchmark case. However, note that the wages alone can represent the utilities in the benchmark case. On the other hand, the supervisor is now other-regarding and the principal has to consider both the agent's and the supervisor's wages at the same time. The supervisor gets the lowest payoffs when she fails to provide productivity levels of environment. Setting $S_3 - \lambda_S(S_3 - W_3)$ lower than $S_4 - \lambda_S(S_4 - W_4)$ reduces the ability of agent with high productivity environment to bribe the successful supervisor in order to misrepresent state 4 as 3. On the other hand, setting $S_3 - \lambda_S(S_3 - W_3)$ too low will result in another coalition in which unsuccessful supervisor persuades the agent in high productivity environment into acting as she is in low productivity state. This is eliminated by making sure that $S_3 - \lambda_S(S_3 - W_3) = S_2 - \lambda_S(S_2 - W_2)$. Finally, the principal sets $S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$ which rewards the successful supervisor and, at the same time, serves to satisfy her participation constraint.

The ranking of agent's utility levels at different states are the same with the benchmark case. However, there is an important difference. In the benchmark case, the principal only considers the monetary payoff and exerted effort level by the agent. When the agent has other-regarding preferences the principal must also think about the sense of inequality between the agent and supervisor. Proposition **??**.c. is a direct consequence of $CIC2$ and the fact that $e_3 = e_4$. Although their total utility is same in those states, the supervisor's and the agent's individual payoffs vary between state 3 and 4: $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$, and $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. Supervisor's monitoring fails at state 3, thus agent has an option to claim that she is working under low productivity environment and generates output with her hard work. In order to prevent this, a higher payoff must be arranged for the agent at state 3 ($W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$). On the other hand, optimal insurance for the agent tells us that her payoff at state 4 should be lower than the one at state 4. This gives a direct incentive to the agent to bribe supervisor when the realized state is 4. Hence, the supervisor must achieve $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$ so that the principal can prevent a collusion between the agent and the supervisor. The difference between state 4 and 3 can be viewed as a cost of obtaining the true information about the state of productivity.

Although the ranking of utilities is not different than the benchmark case, the amount of wages definitely changes according to the intensity of other-regarding preferences. We need to specify an explicit formula for the utility functions in order to find how much, and in which direction the wages will differ from benchmark wages. Thus, it may or may not be beneficial to employ an agent or supervisor with other-regarding preferences depending on the utility functions of the agent and supervisor. Furthermore, we expect that the dispersion between wages at different states can become wider or narrower in comparison with the benchmark case since adjusting wages is one of the tools to cease collusion and their effectiveness may be reduced or enhanced when the parties have other-regarding preferences. The principal may need a further increase or decrease in wages to satisfy collusion constraints with other-regarding preferences.

4 Discussion and Concluding Remarks

Many people act as if they have other-regarding preferences in different economic environments, and concern for another's situation is a motivating factor on their decision making processes in most situations. In this paper, we implement Fehr and Schmidt's (1999) distributional approach for other-regarding preferences into Tirole's (1986) three level hierarchy model. Our aim is to analyze the effect of having other-regarding supervisor or agent on collusive behavior in an organizational hierarchy and on collusion-proof optimal contracts.

Other-regarding preferences affect collusive behavior. Not only it changes the tendency of agent's and supervisor's to offer bribe or take bribe but also it influences the effectiveness of principal's tools she utilizes in preventing collusive actions. Moreover, in case where the supervisor's sensitivity to wage inequality reaches a certain threshold, she does not accept any request of coalition coming from the agent independent of the level of the side transfer. This implies that the role of collusion in organizational hierarchies can be ignored if a supervisor's level of inequity aversion is above the certain threshold.

For the components of optimal contracts, most prominent impact of other-regarding preferences is on effort levels. When the agent is inequity averse principal can exploit this fact to make agent invest higher effort level than she would otherwise. In order to satisfy the participation constraint of supervisor, the effort level induced for agent becomes lower when the supervisor is status seeker, and it is higher when the supervisor is inequity averse. We see more-than-optimal effort when both the supervisor and agent is inequity averse or inequity aversion of the agent dominates competitiveness of the supervisor, less-than-optimal effort when status seeking of the supervisor dominates the inequity aversion of the agent, and optimal effort when these two are equal.

Although the ranking of supervisor's or agent's utilities at different states do not differ between self-interested or other-regarding players, the ranking of their wages is not the same due to social preferences. However, explicit formula of the utility functions are needed to show the exact change in the wages. In addition to the changes in rankings, the dispersion between the wages at different states can become wider or narrower in comparison with the benchmark case. Change in dispersion has consequences for risk averse parties since they do not enjoy the cases in which their possible wages are highly different from their reservation wages.

We show that using other-regarding preferences in the design of optimal collusion-proof contracts generate new insights on the role of collusion in organizations. A question for future research is how the different types of hierarchies interact with other-regarding preferences. This paper investigates the effects of other-regarding parties for only principal-supervisor-agent type of hierarchies, and we think that there is a great scope for implementing other-regarding preferences to other organizational hierarchies as in Bac (1996). Moreover, we take the inequity aversion parameters of parties as observable and common knowledge for simplicity. Additional studies with uncertain inequity aversion parameters seem like an interesting path to follow for further research. Lastly, a further research on this topic is left to be done is adding productive duties to supervisor and making monitoring technology costly as in Bac & Kucuksenel (2006). Note that the principal always tries to change the effort level exerted by the agent since she is the only productive unit in the hierarchy and this limits the options that principal has. Adding effort and effort cost for the supervisor not only increases the chances of principal to satisfy the needs of other-regarding parties but also introduces a possibility of ex-ante collusion between supervisor and agent. Investigating the changes in optimal contracts and collusive actions for this new case may also be worthwhile.

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Appendix

Lagrangian for the solution of principal's problem with other-regarding parties is:

$$L = \sum_i p_i(\theta_i + e_i - W_i - S_i) + \nu(\sum_i p_i V(S_i - \lambda_S(S_i - W_i)) - \overline{V})$$

$$+\mu(\sum_i p_i U(W_i - g(e_i) - \lambda_A(S_i - W_i)) - \overline{U})$$

$$+\gamma(W_3 - g(e_3) - \lambda_A(S_3 - W_3) - W_2 + g(e_2 - \Delta\theta) + \lambda_A(S_2 - W_2))$$

$$+\psi((S_4 - \lambda_S(S_4 - W_4)(1 + 2\lambda_A) + (W_4 - g(e_4) - \lambda_A(S_4 - W_4))(1 - 2\lambda_S) - (S_3 - \lambda_S(S_3 - W_3))(1 + 2\lambda_A) - (W_3 - g(e_3) - \lambda_A(S_3 - W_3))(1 - 2\lambda_S))$$

$$+\pi((S_3 - \lambda_S(S_3 - W_3)(1 + 2\lambda_A) + (W_3 - g(e_3) - \lambda_A(S_3 - W_3))(1 - 2\lambda_S) - (S_2 - \lambda_S(S_2 - W_2))(1 + 2\lambda_A) - (W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2))(1 - 2\lambda_S))$$

Note that we ignore $(CIC1)$, we later are going to show that the solution to the problem satisfies $(CIC1)$.

Taking the derivatives of the Lagrangian above with respect to S_i, W_i, e_i results in following FOCs:

$$\nu V'(S_1 - \lambda_S(S_1 - W_1)) = \frac{1}{1-\lambda_S} + \mu\frac{\lambda_A}{1-\lambda_S}U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1)) \tag{1}$$

$$\nu V'(S_2 - \lambda_S(S_2 - W_2)) = \frac{1}{1-\lambda_S} + \mu\frac{\lambda_A}{1-\lambda_S}U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2)) - \frac{\gamma\lambda_A}{p_2(1-\lambda_S)} + \frac{\pi(1+\lambda_A-\lambda_S)}{p_2(1-\lambda_S)} \tag{2}$$

$$\nu V'(S_3 - \lambda_S(S_3 - W_3)) = \frac{1}{1-\lambda_S} + \mu\frac{\lambda_A}{1-\lambda_S}U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3)) + \frac{\gamma\lambda_A}{p_3(1-\lambda_S)} + \frac{(\psi-\pi)(1+\lambda_A-\lambda_S)}{p_3(1-\lambda_S)} \tag{3}$$

$$\nu V'(S_4 - \lambda_S(S_4 - W_4)) = \frac{1}{1-\lambda_S} + \mu\frac{\lambda_A}{1-\lambda_S}U'(W_4 - g(e_4) - \lambda_A(S_4 - W_4)) - \frac{\psi(1+\lambda_A-\lambda_S)}{p_4(1-\lambda_S)} \tag{4}$$

$$\mu U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1)) = \frac{1}{1+\lambda_A} - \nu\frac{\lambda_S}{1+\lambda_A}V'(S_1 - \lambda_S(S_1 - W_1)) \tag{5}$$

$$\mu U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2)) = \frac{1}{1+\lambda_A} - \nu\frac{\lambda_S}{1+\lambda_A}V'(S_2 - \lambda_S(S_2 - W_2)) + \frac{\gamma}{p_2} + \frac{\pi(1+\lambda_A-\lambda_S)}{p_2(1+\lambda_A)} \tag{6}$$

$$\mu U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3)) = \frac{1}{1+\lambda_A} - \nu\frac{\lambda_S}{1+\lambda_A}V'(S_3 - \lambda_S(S_3 - W_3)) - \frac{\gamma}{p_3} + \frac{(\psi-\pi)(1+\lambda_A-\lambda_S)}{p_3(1+\lambda_A)} \tag{7}$$

$$\mu U'(W_4 - g(e_4) - \lambda_A(S_4 - W_4)) = \frac{1}{1+\lambda_A} - \nu\frac{\lambda_S}{1+\lambda_A}V'(S_4 - \lambda_S(S_4 - W_4)) - \frac{\psi(1+\lambda_A-\lambda_S)}{p_4(1+\lambda_A)} \tag{8}$$

$$\mu U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1))g'(e_1) = 1 \tag{9}$$

$$\mu U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2))g'(e_2) - \frac{(\gamma + \pi(1-2\lambda_S))}{p_2}g'(e_2 - \Delta\theta) = 1 \tag{10}$$

$$\mu U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3))g'(e_3) + \frac{\gamma + (\pi-\psi)(1-2\lambda_S)}{p_3}g'(e_3) = 1 \tag{11}$$

$$\mu U'(W_4 - g(e_4) - \lambda_A(S_4 - W_4))g'(e_4) + \frac{\psi(1-2\lambda_S)}{p_4}g'(e_4) = 1. \tag{12}$$

Proof of Proposition 1: Substituting 5, 6, 7, 8 into 9, 10, 11, 12 gives us $g'(e_1) = g'(e_3) = g'(e_4) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ and $g'(e_2) < \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$. Since $g''(e_i) > 0$, we can rank the efforts as $e_1 = e_3 = e_4 > e_2$. When, $\lambda_A = -\lambda_S$, we have $g'(e_1) = g'(e_3) = g'(e_4) = 1$ and $g'(e_2) < 1$. It means $e_1 = e_3 = e_4 = e^* > e_2$.

When, $\lambda_A < -\lambda_S$, we have $g'(e_2) < g'(e_1) = g'(e_3) = g'(e_4) < 1$. It means $e^* > e_1 = e_3 = e_4 > e_2$. Upper boundary of $g'(e_2)$ goes to $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ in our case. Thus, the principal sets $g'(e_2) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon$ where $\varepsilon > 0$, in order to guarantee maximum output level. Since $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon = g'(e_2) < g'(e_2^B) = 1 - \varepsilon$ we have $e_2 < e_2^B$.

When, $\lambda_A > -\lambda_S$, we have $g'(e_1) = g'(e_3) = g'(e_4) > 1$. This implies that $e_1 = e_3 =$

$e_4 = e^*$. Upper boundary of $g'(e_2)$ also increases to $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ in this case. Thus, the principal sets $g'(e_2) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon$ where $\varepsilon > 0$, in order to get maximum output (profit). Since $g'(e_2) > g'(e_2^B) = 1 - \varepsilon$ we have $e_2 > e_2^B$. For a given ε (where $\frac{\partial \varepsilon}{\partial \Delta\theta} > 0$), when we have $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} = \varepsilon$ we get $g'(e_2) = g'(e^*) = 1$. So, there are some values for λ_A and λ_S where $e_2 = e^*$. Finally, we have $e_2 > e^*$ when $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} > \varepsilon$, and $e_2 < e^*$ when $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} < \varepsilon$.

Q.E.D.

Proof of Proposition 2: First substitute 5, 6, 7, 8 into 1, 2, 3, 4. Then, we have:

$$\nu V'(S_1 - \lambda_S(S_1 - W_1)) = \frac{1 + 2\lambda_A}{1 + \lambda_A - \lambda_S} \tag{13}$$

$$\nu V'(S_2 - \lambda_S(S_2 - W_2)) = \frac{1 + 2\lambda_A}{1 + \lambda_A - \lambda_S} + \frac{\pi(1 + 2\lambda_A)}{p_2} \tag{14}$$

$$\nu V'(S_3 - \lambda_S(S_3 - W_3)) = \frac{1 + 2\lambda_A}{1 + \lambda_A - \lambda_S} + \frac{(\psi - \pi)(1 + 2\lambda_A)}{p_3} \tag{15}$$

$$\nu V'(S_4 - \lambda_S(S_4 - W_4)) = \frac{1 + 2\lambda_A}{1 + \lambda_A - \lambda_S} - \frac{\psi(1 + 2\lambda_A)}{p_4} \tag{16}$$

Now, substitute 1, 2, 3, 4 into 5, 6, 7, 8. Then, we have:

$$\mu U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1)) = \frac{1 - 2\lambda_S}{1 + \lambda_A - \lambda_S} \tag{17}$$

$$\mu U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2)) = \frac{1 - 2\lambda_S}{1 + \lambda_A - \lambda_S} + \frac{\gamma}{p_2} + \frac{\pi(1 - 2\lambda_S)}{p_2} \tag{18}$$

$$\mu U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3)) = \frac{1 - 2\lambda_S}{1 + \lambda_A - \lambda_S} - \frac{\gamma}{p_3} + \frac{(\psi - \pi)(1 - 2\lambda_S)}{p_3} \tag{19}$$

$$\mu U'(W_4 - g(e_4) - \lambda_A(S_4 - W_4)) = \frac{1 - 2\lambda_S}{1 + \lambda_A - \lambda_S} - \frac{\psi(1 - 2\lambda_S)}{p_4} \tag{20}$$

To show that (AIC) is binding, suppose $\gamma = 0$. Then, from 14, 15 and 18, 19, we have:

$$\frac{V'(S_2 - \lambda_S(S_2 - W_2))}{V'(S_3 - \lambda_S(S_3 - W_3))} = \frac{U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2))}{U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3))}. \tag{21}$$

On the other hand, (AIC) implies:

$$W_3 - g(e_3) - \lambda_A(S_3 - W_3) \geq W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2) > W_2 - g(e_2) - \lambda_A(S_2 - W_2) \tag{22}$$

From 17 and 18, we get:

$$S_3 - \lambda_S(S_3 - W_3) > S_2 - \lambda_S(S_2 - W_2) \tag{23}$$

22 and 23 mean that $S_3 - \lambda_S(S_3 - W_3) + W_3 - g(e_3) - \lambda_A(S_3 - W_3) > S_2 - \lambda_S(S_2 - W_2) + W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2)$, i.e. $(CIC3)$ does not bind. This implies that $\pi = 0$. Then,

22 and 24 imply that:

$$W_2 - g(e_2) - \lambda_A(S_2 - W_2) \geq W_3 - g(e_3) - \lambda_A(S_3 - W_3) \tag{24}$$

When we look at 18 and 20, we see that there is a contradiction which completes this part of our proof and shows that $\gamma > 0$, i.e. (AIC) is binding.

Next, suppose second collusion constraint is not binding, i.e. $\psi = 0$.

From 15 and 16, we have $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. $\lambda_A > 0$, so we can write:

$$\frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} > \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} \tag{25}$$

From 15 and 16, we have $S_3 - \lambda_S(S_3 - W_3) \geq S_4 - \lambda_S(S_4 - W_4)$. $\lambda_S < 0.5$, so we can write:

$$\frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} \geq \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} \tag{26}$$

Then 25 and 26 implies that:

$$\frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} > \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} \tag{27}$$

which violates $(CIC2)$. Thus, we must have $\psi > 0$. $(CIC2)$ is binding.

We now look at whether $(CIC3)$ is binding or not. Assume that $(CIC3)$ does not bind, i.e. $\pi = 0$. Then, from 14 and 15, we have $S_2 - \lambda_S(S_2 - W_2) > S_3 - \lambda_S(S_3 - W_3)$.

We know that (AIC) is binding, so $(CIC3)$ can be represented as $(CIC3') : S_3 - \lambda_S(S_3 - W_3) \geq S_2 - \lambda_S(S_2 - W_2)$. Now, with the result in the paragraph above, we can say that when $\pi = 0$ $(CIC3)$ is violated. Therefore, $\pi > 0$ and $(CIC3)$ is binding.

With the proof of proposition 3, we are going to show that $(CIC1)$ is already satisfied with the current solution and hence $(CIC1)$ is not binding.

Q.E.D.

Proof of Proposition 3: We know that both (AIC) and $(CIC3)$ is binding. This implies that $S_2 - \lambda_S(S_2 - W_2) = S_3 - \lambda_S(S_3 - W_3)$. Moreover, from 13, 14 and 16, we have $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$. Therefore, the ranking of supervisor's utilities at different states is $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2) = S_3 - \lambda_S(S_3 - W_3)$. From 17, 18 and 20, we have

$$W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2) \tag{28}$$

We know that $(CIC2)$ binds and $g(e_3) = g(e_4)$. This implies that

$$\frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} = \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A}.$$

Since $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$, we have $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. This completes the proof for $W_3 - g(e_3) - \lambda_A(S_3 - W_3) >$

$W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$.

Now, it is quite straightforward to show that $(CIC1)$ is already satisfied and not binding, since $S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$ and $W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$.

Q.E.D.