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Abstract

This study scrutinizes the implications of a vendor bid in an open ascending auction with a seller of an indivisible good and many potential buyers. The seller can set a reserve price, and both the seller and the bidders have private signals and interdependent values. We show that no strictly increasing reserve price function exists in the presence of a vendor bid. We also show that a vendor bid must be large enough to be credible, and thus vendor bids may not be used in equilibrium. The vendor will exercise her vendor bid option if and only if her private signal is large enough.

Keywords: Vendor Bid; Interdependent Valuation; Open Ascending Auction
JEL Classification: D44

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1 Introduction

In this paper, we scrutinize auctions in which the seller (or the vendor) may actively participate in the auction and compete against a bidder via a *vendor bid* under some conditions. Vendor bids are allowed in auctions of many different goods; see, for example, real estate auctions. Similar to eBay auctions, the seller may set a reserve price before the auction starts. If any bid meets the reserve price, standard auction rules prevail afterwards. If, however, there is no bid that meets the reserve price, the seller may submit a vendor bid that will be announced by the auctioneer. While many authors like Klemperer (2004), Krishna (2002) and Milgrom (2004) have studied different auction designs and their revenue and efficiency implications, the literature has been silent on auctions that allow vendor bids. This paper fills this gap.

To scrutinize the role of a vendor bid, we focus on auctioning an indivisible good by an open ascending (English) auction with interdependent (affiliated) private values. In order to be as general as possible, we employ a model that is similar to Milgrom and Weber (1982). The difference is that the seller can set a reserve price and will have the opportunity to compete against the highest bidder by submitting a vendor bid if the highest bid has not met the reserve price. Our results show that, in equilibrium, a vendor bid option does not allow any strictly increasing reserve price function to exist. Furthermore, we find that any sequential equilibrium will imply a prohibitively large reserve price. Vendor bids also come with a cost for the seller: given a prohibitively large reserve price, the bidder will correctly anticipate that some seller types may prefer to sell at the price when the second-highest bidder has dropped out. Since values are affiliated, these sellers could try to mimic a larger valuation than they have, and consequently, the vendor bid has to be substantially larger than the last bid to make it credible.

A number of studies in auction theory have investigated a seller's incentive to actively participate in the auction by submitting a bid. This is usually done secretly by submitting a "shill" bid; see, inter alia, Lamy (2007); Chakraborty and Kosmopoulou (2004); Vincent (1995); Graham et al. (1990); and Ashenfelter (1989). The effectiveness of using a shill bid is not unanimously established. Chakraborty and Kosmopoulou (2004), and Lamy (2007) find that shill bidding reduces the probability of sale and makes the seller and the bidders worse off. Alternatively, Graham et al. (1990) show that shill bidding can increase the revenue in an English auction for an independent private values model with distributionally heterogeneous bidders. The seller, in their model, can adopt a reserve price which is a function of the observed bids or use a phantom bid. In a similar paper, Lopomo (2000) shows in a model similar to the one in Milgrom and Weber (1982) that the seller's expected profit is maximized in a symmetric equilibrium of an English auction when the seller can place a reserve price after all but one buyer has dropped out of the auction. In a common value second-price auction setting, Vincent (1995) shows that the seller might be better off by asking the auctioneer not to reveal the reserve price, and have the auctioneer create bids off the wall if the auction price is not high enough.

Creating bids off the wall or shill bidding is usually not legal. Even if such a strategy is used in auctions, phantom bids do not change the bidding strategies of prospective buyers. By contrast, vendor bids are legal and observed (as the auctioneer is required to make a public announcement when a vendor bid is submitted).[1] Given the prevalence of auctions with vendor bids, it is surprising that there is no study that systematically scrutinizes the economic implications of employing vendor bids, and thus our paper fills an important gap in the literature.[2]

The remainder of the paper is structured as follows. In the following section, the model is introduced, the optimal bidding strategies are derived, and the model is solved for an optimal secret reserve price. In Section 3, the model is solved for the optimal vendor bids. Section 4 concludes. For convenience, most of the proofs are relegated to the Appendix.

2 The model

In our model, there is an indivisible good for sale. The good is owned by a single seller (a vendor) who has a private signal for the value estimate of the good. There are many potential buyers. Each potential buyer also has a private signal for the value estimate of the good. The vendor runs an open ascending (English) auction. The model is similar to Milgrom and Weber (1982) such that both the seller and the bidders have private signals, and all bidders' private signals are affiliated making their valuations interdependent. The difference is that the seller can set a reserve price. If this price is met by at least two

[1]Of course, if all serious bidders can identify a shill bidder, and this shill bidder has the same objective as the vendor, vendor and shill bids are strategically equivalent

[2]Our paper can be related also to a small but growing strand of the literature on real-estate auctions (Chow et al., 2015; Stevenson and Young, 2015; McAfee et al., 2002; Quan, 1994). This strand of the literature (perhaps, except for McAfee et al., 2002) is mainly empirical, focusing explicitly on housing auctions. The main questions that this strand of the literature addresses are on the comparison of auctions versus private party sales (Chow et al., 2015), on offering secret or observed reserve prices (Stevenson and Young, 2015), on the comparison of sealed-bid versus open outcry auctions (Chow and Ooi, 2013); and on identifying the optimal reserve price (McAfee et al., 2002).

bidders, the auction turns out to be a standard ascending auction with private (affiliated) values such that the good sells to the highest-bidder at a price at which the second-highest bidder will have dropped out of the auction. If, however, this price is not met by the second-highest bidder, then the seller can either accept the second-highest bid (the seller can sell the item to the highest-bidder at a price at which the second-highest bidder has dropped out of the auction) or she can make a vendor bid. If the seller makes a vendor bid, then the highest bidder and the vendor run a similar ascending auction that starts at the first vendor bid. Note carefully that we make no assumption when the reserve price will be revealed to the bidders. The reason that we will be able to show that the timing of revelation does not play any role for our analysis. Hence, we will demonstrate that it will not matter whether the reserve price will be announced before the auction starts, or when the reserve price is reached (or at any other time in between). In particular, we consider a game that has potentially four stages as outlined by Table 1.

Table 1: Game structure

Stage I: The vendor sets a reserve price.
Stage II: An (English) ascending-bid auction is run.
Stage III: If the second-highest bid is above the reserve price, the items sells to the highest bidder for a price equal to the second-highest bid. If the second-highest bid is below the reserve price, the vendor either accepts this price or makes a vendor bid.
Stage IV: In case of a vendor bid, the highest bidder and the vendor run an (English) ascending auction that starts at the first vendor bid. If the vendor drops out, the item sells for the price where the vendor has dropped out. If the bidder drops out, the item remains unsold.

The utility of bidder i depends on his private signal σ_i and the private signals of all other bidders σ_{-i} and the private signal of the vendor s:

$$V_i = v(\sigma_1, \cdots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \cdots, \sigma_n, s); \quad v_s > 0; \quad \forall j : v_{\sigma_j} > 0; v_{\sigma_j s} \leq 0. \tag{1}$$

The utility of the seller depends on all private signals of the bidders and her own signal and is given by

$$U = u(\sigma_1, \cdots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \cdots, \sigma_n, s); \quad u_s > 0; \quad \forall j : u_{\sigma_j} > 0; u_{\sigma_j s} \leq 0. \tag{2}$$

Note that $v_{\sigma_j s} \leq 0$ and $u_{\sigma_j s} \leq 0$ imply that the positive marginal effect of a positive signal is not reinforced by an increase in another signal. This does not mean that signals cannot be affiliated. On the contrary, we assume that all signals of the bidders are drawn from the joint distribution of signals $\Phi(\sigma_1, \cdots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \cdots, \sigma_n)$, over support $\sigma_j \in [\underline{\sigma}, \overline{\sigma}]$, $\forall j = \{1, \cdots, n\}$, that allows signals to be positively affiliated. However, we assume that the vendor signal and any bidder signal are not affiliated as the vendor's information is on different characteristics than the bidders'. The vendor signal s is distributed over support $[\underline{s}, \overline{s}]$ according to the pdf $G(s)$. We assume that the expected utility of a bidder is larger than the expected utility of the vendor, so that the auction makes sense to begin with. Furthermore, $\overline{V} = v(\overline{\sigma}, \cdots, \overline{\sigma}, \overline{s})$ denotes the maximum utility of a bidder.

Let $\Sigma_j = (\sigma_j, \cdots, \sigma_n)$ denote the vector of all signals that have been revealed through bidders dropping out from the auction at certain prices, such that p_k is the price that has made bidder k stop bidding. Prices and bidders are ranked such that $p_k > p_{k+1}$. Let $r(s)$ denote the reserve price as set optimally by a vendor of type s. The optimal bidding strategy is given by

$$\beta_i = v(\sigma_i, \cdots, \sigma_i, \Sigma_j, \hat{s}), \tag{3}$$

where signal σ_j is revealed by bidder j dropping out at price

$$p_j = \beta_j = v(\sigma_j, \cdots, \sigma_j, \Sigma_{j+1}, \hat{s}). \tag{4}$$

In equations (3) and (4), $\hat{s}$ is the expected signal of the vendor as perceived by bidders, and has to be updated according to Bayes' Theorem. If the reserve price function were strictly monotonically increasing (i.e., if $r'(s) > 0$, $\forall s \in [\underline{s}, \overline{s}]$), an inverse reserve price function $\rho(p) = r^{-1}(p)$ with $\rho'(p) > 0$ would exist, and $\rho(p)$ would indicate the type of the seller if the reserve price were met at price p. The following result, however, shows that the option of a vendor bid does not allow the reserve price function to be strictly increasing.

Proposition 1 *(i) In equilibrium, a strictly increasing reserve price function such that $r(s) < \overline{V}$ and $r'(s) > 0$, $\forall s \in [\underline{s}, \overline{s}]$, does not exist. (ii) Any sequential equilibrium implies a prohibitively large reserve price, that is, $r(s) \geq \overline{V}$, $\forall s \in [\underline{s}, \overline{s}]$.*

Proof. See Appendix A.1. ■

The reason is that a vendor bid allows a vendor to continue the auction and thus to avoid the threat of not selling. A strictly monotonically increasing reserve price function can exist only if different types have different exposure to the risk of not selling. Since all types may continue with a vendor bid, all types have also no additional cost in mimicking a type that is stronger than their true type in order to increase the bids. Consequently, all sellers will set the same reserve price.

Proposition 1 also shows that irrespective of the private signal that a seller receives, all sellers set a prohibitively high reserve price is any sequential equilibrium.[3] If there is zero probability to meet the reserve price, Bayesian updating will play no role, and potential buyers will stick to their priors such that they consider the average of the private signals as the seller's private signal. If, however, the reserve price was ever met, then they would believe that the reserve price must have been set by a seller with the lowest private signal leading them to update their priors. Consistent with these beliefs, if a seller, irrespective of her private signal, had set a reserve price such that there was some positive probability that the reserve price could have been met, then the bidders would have updated their beliefs and would have treated the seller as one having received the lowest signal. As the values are positively affiliated, such an update of beliefs reduces the value estimates and hurts the seller. Thus, to avoid this, a sequentially rational strategy that is consistent with bidders' beliefs is to set a reserve price such that it cannot be met with a positive probability.

These results suggest that, without loss of generality, we can now assume that all types will set a prohibitively high reserve price, and we can scrutinize the optimal vendor bids under this assumption. In any case, the vendor bid is able to undo the risk of not selling, as the auction is not over if the reserve price is not met. Thus, the vendor bid works like a contingent reserve price that is set after the second-highest bidder has dropped out from the auction. It is thus not surprising that the reserve price option is never exercised if a vendor bid is possible.

3 Optimal vendor bids

The analysis now is continued under the assumption that all types set a prohibitively high reserve price such that $r(s) \geq \overline{V}, \forall s \in [\underline{s}, \overline{s}]$. Once the second-highest bidder has stopped

[3] Appendix A.1 shows also that other perfect Bayesian Nash equilibria exist that do not qualify as a sequential equilibrium.

bidding, the highest bidder will stop as well, as she is aware of the vendor bid option and knows that she never wants to reach the reserve price. Let $F_1(p,x)$ denote the cumulative distribution of the valuations of the highest bidder if the price p has been reached. When the second-highest bidder drops out, it is given by $F_1(\underline{\beta}_1, p)$ where $\underline{\beta}_1$ denotes the lowest valuation of any highest bidder.[4] $F_1(p,x)$ follows from Bayes' Rule such that

$$F_1(p,x) = \frac{F_1(\underline{\beta}_1, x)}{1 - F_1(\underline{\beta}_1, p)}; \quad f_1(p,x) = \frac{\partial F_1(p,x)}{\partial x} = \frac{f_1(\underline{\beta}_1, x)}{1 - F_1(\underline{\beta}_1, p)}$$

holds. Note carefully that the start of the auction conveys information to the vendor on the highest bidder's type: if the highest bidder has not dropped out at p, the vendor will update her belief about the expected signal of the bidder. Let $\hat{\sigma}_1(p)$ denote the vendor's expectation of the highest bidder's signal, and let $\gamma(\cdot)$ denote the inverse bidding function of the highest bidder in the (follow-up) auction that starts with the vendor's bid. Then,

$$\begin{aligned} \hat{\sigma}_1(p) &= \gamma\left(\int_p^{\overline{\beta}_1} x f_1(p,x)dx\right) = \gamma\left(\frac{1}{1 - F_1(\underline{\beta}_1, p)}\left(\int_p^{\overline{\beta}_1} x f_1(\underline{\beta}_1, x)dx\right)\right), \qquad (5) \\ \frac{d\hat{\sigma}_1(p)}{dp} &= \frac{\gamma'(\cdot) f_1(\underline{\beta}_1, p)}{1 - F_1(\underline{\beta}_1, p)}\left(\int_p^{\overline{\beta}_1} x f_1(p,x)dx - p\right) > 0, \end{aligned}$$

where $\overline{\beta}_1$ denotes the maximum utility of the highest bidder. Expression (5) shows that the expected signal will increase with an ongoing auction, and this effect increases the expected utility of the vendor as an ongoing auction indicates to the vendor that the highest bidder has received a good signal.

Suppose that the vendor has started the auction with the highest bidder and the bidder has not yet dropped out. We will be more precise on how large the starting vendor bid should be. But it should be clear that the vendor is always better off by starting with the lowest possible bid and increase it. When would the vendor want to continue bidding? At price p the vendor's payoff is given by $[1 - F_1(p,b)][b - u(\hat{\sigma}_1(\cdot), s)]$ if he continues bidding such that $b \geq p$. He would like to continue if the marginal gain from continuing bidding is positive, that is if

$$1 - F_1(p,b) - f_1(p,b)(b - u(\hat{\sigma}_1(p), s)) \geq 0 \text{ for } b \geq p.$$

The optimal bidding behavior is determined by the stopping rule

$$1 - f_1(b^*, b^*)(b^* - u(\hat{\sigma}_1(b^*), s)) = 0. \qquad (6)$$

[4]Note the sloppiness in our notation. F_1 will also depend on all signals revealed so far such that $F_1(p, x, \beta_{-1})$. To simplify the notation, we express this as $F_1(p,x)$.

Note that $F_1(b^*, b^*) = 0$. The optimal bidding behavior leads to an optimal vendor bid function $b^*(s)$, and it is straightforward to show that $b^{*\prime}(s) > 0$. Note that expression (6) is derived under the assumption that all types will make vendor bids. The problem is now that not all types will want to make a vendor bid, but will (optimally) accept the second-highest bid. This will happen if

$$1 - f_1(\beta_2, \beta_2)(\beta_2 - u(\hat{\sigma}_1(\underline{\beta}_1), s)) \leq 0. \tag{7}$$

Note carefully that we have distinguished between the second-highest bid β_2 and the lowest valuation of the highest bidder, $\underline{\beta}_1$, given that universal vendor bids will be made as $F_1(\cdot)$ uses $\underline{\beta}_1$ as the lower bound. Why do we have to distinguish between β_2 and $\underline{\beta}_1$? As some vendor types will accept the second-highest bid, the highest bidder faces an adverse selection problem. If she had assumed that all types would make a vendor bid, she would have overestimated the expected signal when the item was sold for the price of the second-highest bid. She will anticipate this effect, and therefore, her optimal bidding behavior before any (credible) vendor bid has been made is given by expression (3) with

$$\hat{s} = \int_{\underline{s}}^{s'} s dG(s) < s'$$

where $s' : \underline{s} < s' < \overline{s}$, is the worst type who can credibly make a vendor bid. How is s' determined? In general, any vendor can mimic to be of another type, say type $\tilde{s}$, and behave as if $\tilde{s}$ is expected to behave, that is, make vendor bids until $b^*(\tilde{s})$ is reached. Hence, $\forall s \in [\underline{s}, s'], \forall \tilde{s} \in [s', \overline{s}]$, the no-mimicking condition

$$\beta_2 - u(\hat{\sigma}_1(\underline{\beta}_1), s) \geq \left(1 - F_1(\underline{\beta}_1, b^*(\tilde{s}))\right)(b^*(\tilde{s}) - u(\hat{\sigma}_1(b^*(\tilde{s})), s)) \tag{8}$$

must hold. It says that no vendor of type s should have an incentive to follow the optimal bidding behavior of a vendor of any type $\tilde{s}$. The LHS has the realization of the second-highest bid; and the RHS is the expected utility of a vendor of type s pretending to be of type $\tilde{s}$. It can easily be seen that inequality (8) can never be fulfilled for $\tilde{s} = \underline{s}$: bidders would have to believe that all vendors make a bid, so $\beta_2 = \underline{\beta}_1$, leading not only to the winner's curse, but enabling low vendor types to increase the selling price. Hence, the start of a vendor bid must be such that it implies a cost for better types that is smaller than the cost for worse types. This can only be guaranteed if the auction between the highest bidder and the vendor starts with a price where the acceptance probability of the highest bidder is not too large.

We can identify the range of such credible vendor bids:

Proposition 2 *Mimicking any type $\tilde{s} > s'$, where $s' : \underline{s} < s' < \overline{s}$, can never be optimal, and if the no-mimicking condition* (8) *is fulfilled for $s = \underline{s}$, it is fulfilled for all $s \in [\underline{s}, s']$.*

Proof. See Appendix A.2. ∎

Proposition 2 shows that any vendor of type $s \in [\underline{s}, s']$ cannot make a credible vendor bid, but will accept the second-highest bid, while vendors of type $s \in [s', \overline{s}]$ can make credible bids and the bidders will not be mistaken in the vendor reporting truthfully as at $s = \tilde{s} = s'$, the maximized defections profits coincide with the optimal stopping rule.

4 Concluding remarks

The literature on auction theory has not yet paid attention to the economics of vendor bids, although a vendor bid is legal and has been prevalent in many different auctions. To fill this gap in the literature, we have studied the implications of employing a vendor bid in auctioning an indivisible good. We have focused on an open ascending auction with a single seller and many potential buyers, all of which receive private signals and have interdependent value estimates of the item for sale. The seller is given the option to set a secret reserve price and to compete against the highest bidder by submitting a vendor bid - that will be announced by the auctioneer - when the second-highest bid is below the reserve price. We have first scrutinized whether the seller with a given private signal would have an incentive to set a secret reserve price, and have shown there is no such strictly increasing optimal reserve price, although a prohibitively large reserve price for all types emerges in a sequential equilibrium. We then have studied the optimal bidding behavior, derived a stopping rule for the bidders and a no-mimicking condition for the seller that determines the range of credible vendor bids. We have found that a credible vendor bid exists only for a subset of sellers with higher private signals. Thus, even if an auction format allows for a vendor bid, it may well happen that the vendor will not exercise this option. Our analysis may suggest that the vendor bid option will make a vendor always better off in terms of expected revenues. This conclusion, however, is not valid in general. While a vendor bid undoes the risk of not selling, it also implies that the highest bidder will never increase her bid if the reserve price has not yet been met. Without vendor bid, if a strictly increasing reserve price function exists, the highest bidder is prepared to bid up to her valuation. Furthermore, increasing reserve price functions allow strong vendor types to signal their type to bidders. Any revenue comparison will thus also depend on the distribution of vendor types.

Appendix

A.1 Proof of Proposition 1

(i) We can do the proof by contradiction. Suppose there exists a strictly monotonically increasing reserve price function in the relevant range of the utilities of the bidders. Consider, then, any seller of type $s < \overline{s}$ that is supposed to set a reserve price of size $r(s) < r(\overline{s})$. Let $\hat{\beta}_2$ denote the expected second-highest bid. If the second-highest bid is below the reserve price, then the seller either accepts this bid anyway, or starts a new ascending-bid auction with the highest bidder, and makes a vendor bid as the starting price. The optimal bidding strategy depends on when the reservation price will be revealed. If it is revealed before the auction starts, the optimal bidding behavior is given by

$$\beta_i = v(\sigma_i, \cdots, \sigma_i, \Sigma_j, s), \tag{A.1}$$

where β_i denotes the maximum bid after which the bidder drops out when already j bidders have dropped out. If the reserve price is disclosed once it has been reached, the maximum bid is given the implicit function

$$\beta_i = v(\sigma_i, \cdots, \sigma_i, \Sigma_j, \rho(\beta_i)), \tag{A.2}$$

as long as the the reserve price had not yet been met.[5] Once it is met, bidding behavior is also given by (A.1). The same is true if the reservation price is revealed some time after the auction begins and before the reservation price is reached.

Given that the utility of a bidder is an increasing function of the private signal of the vendor, and that ρ is the inverse reserve price function with $\rho' > 0$, if each bidder expects the reserve price to be set according to a monotonically increasing reserve price function, a reserve price not being met will lead to an increase in β_i. But then all vendor types have an incentive to set $r = r(\overline{s})$ so as to increase the expected second-highest bid $\hat{\beta}_2$ contradicting $r(s) < r(\overline{s})$ for any $s < \overline{s}$.

(ii) Since no strictly increasing reserve price function exists, all vendors will set the same reserve price. It is then relatively straightforward to show that any reserve price $r \in [r^*(\overline{s}), \infty[$, where $r^*(\overline{s})$ denotes the optimal reserve price of type $\overline{s}$ without a vendor bid option, can qualify as a perfect Bayesian Nash equilibrium: all bidders expect all types to set a reserve price r, and if the reserve price is met below r or not met when bids reach r they update their beliefs such that the deviating type is a vendor that has received a signal of $\underline{s}$. However, only a prohibitively large reserve price can establish a sequential equilibrium as no strictly mixed strategies exist that converge to any equilibrium in which the reserve price is not prohibitively large. The reason is that the sequential equilibrium does not allow any knife-edge property such that beliefs jump close to both sides of the reserve price. To support a reserve price that is not prohibitively large, however, it is

[5] A required property is that $v_s(\cdot)\rho'(\cdot) < 1$ holds such that the RHS of (A.2) does not increase stronger than its LHS.

necessary that the belief update below and above the candidate reserve price warrants a jump in the belief structure.

Only a prohibitively large reserve price can fulfill the requirement of a sequential equilibrium (SE): As above, consider the following belief system: unless the reserve price is met (i.e., so long as $r(s) \geq \overline{V}, \forall s \in [\underline{s}, \overline{s}]$), the bidders continue using the average of the private signals that the seller can receive, that is, $\hat{s}$. If, however, a reserve price has been reached (i.e., if $r(s) < \overline{V}, \forall s \in [\underline{s}, \overline{s}]$), then the bidders update their priors such that the reserve price must have been set by the vendor that has received a signal of $\underline{s}$. For an SE with a candidate equilibrium strategy $r(s) = \overline{V}, \forall s \in [\underline{s}, \overline{s}]$ we need strictly mixed strategies that converge to the SE. Consider the following strictly mixed strategies of a seller with any private signal: set $r(s) = \overline{V}$ with probability $\lambda_n = 1 - (1/n)$; and set $r(s) > \overline{V}$ with the complementary probability $1 - \lambda_n = (1/n)$, where $n \in \mathbb{N}$. It is now straightforward to show that $\lim_{n \to \infty} \lambda_n = 1$, that is, a sequence exists that converges the candidate equilibrium, and that the (constant) sequence of beliefs - consistent with the belief structure - converges to $\hat{s}$. Since the strategies are optimal given beliefs and the beliefs are derived from Bayes' Rule for the optimal strategies, and there exists a sequence of strictly mixed strategies and beliefs converging to the candidate equilibrium, $r(s) = \overline{V}$ can be supported in a sequential equilibrium. Obviously any $r(s) > \overline{V}$ can be supported in a similar fashion, whereas any reserve price $r \in [r^*(\overline{s}), \overline{V})$ qualifying as a perfect Bayesian Nash equilibrium cannot be supported in a sequential equilibrium.

A.2 Proof of Proposition 2

We can rewrite the no-mimicking condition given by equation (8) such that

$$\psi(\cdot) = [\beta_2 - u(\hat{\sigma_1}(\underline{\beta}_1), s)] - \left(1 - F_1(\underline{\beta}_1, b^*(\tilde{s}))\right)(b^*(\tilde{s}) - u(\hat{\sigma_1}(b^*(\tilde{s})), s)), \tag{A.3}$$

where the first-order derivative w.r.t true type s for any given $\tilde{s}$ is

$$\psi_s(\cdot) = -\frac{\partial u(\hat{\sigma_1}(\underline{\beta}_1), s)}{\partial s} + (1 - F(\cdot))\frac{\partial u(\hat{\sigma_1}(b^*(\tilde{s}), s)}{\partial s} < 0.$$

The negative sign of $\psi_s(\cdot)$ follows from the negative cross-derivative of $u_{\sigma_j, s}$, given by equation (2), and from increasingness of signals with increasing prices in an ongoing auction, given by equation (5). That is, we know from equation (5) that $\hat{\sigma_1}(b^*(\tilde{s})) > \hat{\sigma_1}(\underline{\beta}_1)$ as $b^*(\tilde{s}) > \underline{\beta}_1$, and from equation (2) (along with equation (5)) that

$$\frac{\partial u(\hat{\sigma_1}(b^*(\tilde{s}), s)}{\partial s} \leq \frac{\partial u(\hat{\sigma_1}(\underline{\beta}_1), s)}{\partial s}.$$

This gives us $\psi(s', \cdot) = 0$ for any $\tilde{s}$. At the same time, the optimal defection is maximizing the defection payoff given that the bidder believes in the vendor truthfully reporting, which will be mutually consistent only for $\tilde{s} = s'$ as is implied by the first-order condition above, that is, for all lower s, the marginal payoff is negative. Moreover, we can show that the optimal stopping rule given by equation (3) is fulfilled at s':

$$1 - f_1(b^*(s'), b^*(s'))(b^*(s') - u(\hat{\sigma}_1(b^*(s')), s')) = 0.$$

that is, at $s = \tilde{s} = s'$, the maximized defection payoff coincides with the optimal stopping rule, and thus, the bidder is not mistaken in the vendor reporting truthfully. Therefore, $\tilde{s} = s'$ and $\psi(s', s') = 0$ must hold. Due to the first-order condition above, it is also clear that mimicking any type $\tilde{s} > s'$ can never be optimal.

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